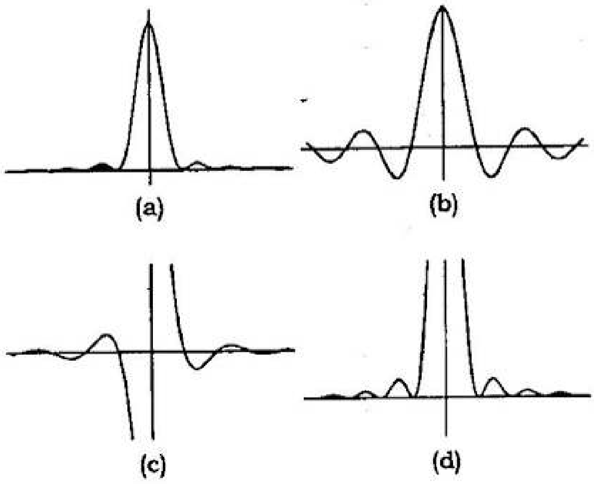
4. Which of the following graphs gives the best representation of the real-valued function $y(x) = \frac{\sin x}{x}$ vs. x?



5. A particle of mass m moves along the x-axis under the influence of the potential $V(x) = V_0(e^{-2\alpha x} - 2e^{-\alpha x})$. If the particle oscillates with a small amplitude around the minimum of the potential, what is the period of oscillation? Take $m = 4$, $V_0 = 2$ and $\alpha = \frac{3}{4}$ (in appropriate units).

(a) 0·12

(b) 1·33

(c) 8·37

(d) 11·17

6. A simple pendulum, made of a point mass m attached to a massless string of length l, oscillates with a time period T_1. A second pendulum, in the shape of a thin uniform rod, has the same length l and the same total mass m and oscillates with the time period T_2. The thickness of the rod is negligible compared to l. The value of the ratio (T_2 / T_1) is

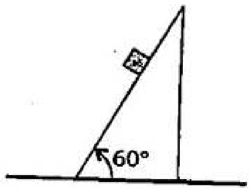
(a) 0·44

(b) 0·67

(c) 0·82

(d) 1·00

7. A block of mass 1 kg is initially held at rest on the frictionless surface of a wedge of mass 5 kg (see figure). The wedge itself lies on a horizontal frictionless surface. The block is now released so that it is allowed to slide down the slope. How far down the slope does the block move when the wedge has moved a distance of 1 cm?



(a) 12·0 cm

(b) 10·0 cm

(c) 8·0 cm

(d) 6·9 cm

8. A planet of mass m moves in an orbit around the Sun (of mass M). The nearest and farthest distances from the Sun during the motion are a and b, respectively and G is the gravitational constant. The magnitude of the angular momentum of the planet around the Sun is [Hint : You may use conservation laws for energy and angular momentum.]

(a) $m\sqrt{2GM(a+b)}$

(b) $m\sqrt{2GM}(ab)^{1/4}$

(c) $m\sqrt{2GM\left(\frac{ab}{a+b}\right)}$

(d) $m\sqrt{2GM\left(\frac{a^2+b^2}{a+b}\right)}$

9. A truck is moving along a straight highway with a speed of 108 km/hr towards a source (fixed on the highway) of microwave radiation of frequency 10 GHz. What will be the beat frequency if the radiation reflected by the truck is superposed with the emitted radiation?

(a) C kHz

(b) 1 kHz

(c) 2 kHz

(d) 10 GHz

10. Consider the following four functions each representing a plane wave :

I. $f_1(x, y, z, t) = \sin(\sqrt{222}\, t - 2x - 7y + 13z)$

II. $f_2(x, y, z, t) = \sin(\sqrt{14}\, t - 2x - 3y - z)$

III. $f_3(x, y, z, t) = \sin(\sqrt{89}\, t - 4x - 3y + 8z)$

IV. $f_4(x, y, z, t) = \sin(\sqrt{125}\, t + 5x - 6y + 8z)$

Which of the following pairs represents waves moving in mutually perpendicular directions?

(a) I and III

(b) I and IV

(c) II and III

(d) II and IV

11. What is the maximum amount of work that can be extracted from an object of heat capacity C_p, initially at a temperature T_2, by a heat engine that operates between this object and a reservoir at temperature T_1?

(a) $C_p(T_2 - T_1)$

(b) $C_p(T_2 - T_1) - C_p T_1 \ln(T_2 / T_1)$

(c) $C_p(T_2 - T_1) \ln(T_2 / T_1)$

(d) $C_p(T_2 + T_1)/2$

12. Three concentric thin spherical shells of radii a, b and c ($a < b < c$) carry uniform surface electric charge of densities σ, $-\sigma$ and σ, respectively. The electric potential at the surface of the outermost shell is

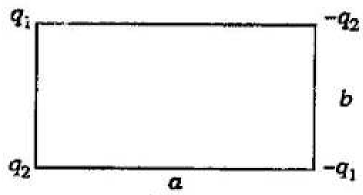
(a) $\frac{\sigma}{\varepsilon_0}(c - b + a)$

(b) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{a} - \frac{b^2}{a} + a\right)$

(c) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{b} - b + \frac{a^2}{b}\right)$

(d) $\frac{\sigma}{\varepsilon_0}\left(c - \frac{b^2}{c} + \frac{a^2}{c}\right)$

13. Four point charges $\pm q_1$ and $\pm q_2$ are placed at the corners of a rectangle of sides a and b as shown in the figure :



What is the magnitude of the dipole moment of the system?

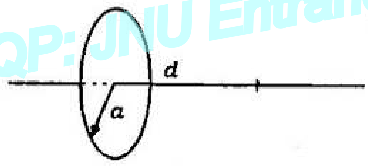
(a) $(q_1 + q_2)\sqrt{a^2 + b^2}$

(b) $(q_1 - q_2)(a - b)$

(c) $\sqrt{(q_1 + q_2)^2 a^2 + (q_1 - q_2)^2 b^2}$

(d) The dipole moment will depend on the choice of origin

14. A very thin circular wire of radius a carries electric charge of uniform linear density ρ. On its axis, the magnitude of the electric field attains its maximum value at a perpendicular distance d from the plane of the wire (see the figure below). The value of d is



(a) 0

(b) $0 \cdot 32a$

(c) $0 \cdot 50a$

(d) $0 \cdot 71a$

15. An electron with kinetic energy of 10 keV is moving in a circular orbit in a plane perpendicular to a uniform magnetic field of strength 2 tesla $(= 2 \times 10^4$ gauss). The period of rotation is nearest to

(a) $2 \cdot 8 \times 10^{-12}$ s

(b) $5 \cdot 6 \times 10^{-12}$ s

(c) $1 \cdot 8 \times 10^{-11}$ s

(d) $1 \cdot 1 \times 10^{-10}$ s

6/K5

16. Consider a region of space in which there are uniform electric and magnetic fields which are mutually parallel. A charged particle released from rest in this region will move in a

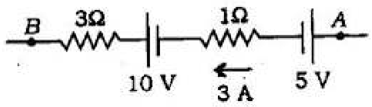
(a) straight line

(b) circle

(c) helix

(d) cycloid

17. A current of 3 amperes flows from A to B through the circuit shown below. The potential difference $V_A - V_B$ is



(a) +1 V

(b) +7 V

(c) −11 V

(d) +27 V

18. The peak value of the charge on the capacitor in an L-C oscillator is 10^{-5} coulomb. The values of L and C are 10^{-2} henry and 10^{-6} farad, respectively. The maximum value of energy stored in the inductor, during the oscillation, is nearest to

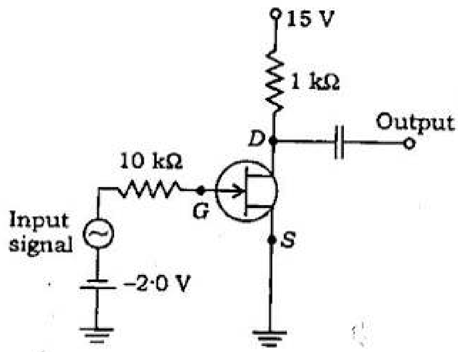
(a) 10 J

(b) 5 J

(c) 2×10^{-3} J

(d) 5×10^{-5} J

19. A JFET amplifier has $I_{DSS} = 16$ mA and a pinch-off voltage $V_P = -4$ V. If it is used in the circuit shown in the figure below, find the value of voltage amplification for small signals :



(a) −4·0

(b) −4·5

(c) −7·5

(d) −10·0

20. The isothermal compressibility $-\frac{1}{V}\frac{\partial V}{\partial p}$ of one mole of van der Waals gas as a function of temperature and volume is (a and b are constants in the van der Waals equation of state)

(a) $V^2(V-b)/(k_B TV^2 - a(V-b))$

(b) $V(V-b)^2/(k_B TV^2 - 2a(V-b))$

(c) $V^2(V-b)^2/(k_B TV^3 - 2a(V-b)^2)$

(d) $(V-b)/k_B T$

21. The intensity of radiation emitted by the Sun has its maximum value at a wavelength of 500 nm. The corresponding value for the star Polaris is 380 nm. Assuming that these stars behave like blackbodies, the ratio (T_S/T_P) of the surface temperatures of the Sun and Polaris is

(a) 1·70

(b) 1·32

(c) 0·76

(d) 0·58

22. A small block of copper with a specific of 100 calories/°C is initially at a temperature of 540 °C. It is then dropped into a large tank of water maintained at 27 °C. What is the change in entropy of the combined system after equilibrium is established?

(a) Decreases by 100 calories/K

(b) Increases by 171 calories/K

(c) Decreases by 270 calories/K

(d) Increases by 70 calories/K

23. Let v_x, v_y and v_z denote the components of the velocity along x-, y- and z-directions, respectively, of an ideal gas particle. At the absolute temperature T, the average value of the product $v_x^2 v_y^2 v_z^2$ is proportional to

(a) T

(b) $T^{3/2}$

(c) T^3

(d) T^6

24. An optical telescope of diameter 2 m is being used to observe stars. What is the order of magnitude of the minimum angular separation between two stars that can be resolved by this telescope?

(a) 10^{-5}

(b) 10^{-7}

(c) 10^{-9}

(d) 10^{-11}

25. According to the uncertainty principle, the kinetic energy of an electron confined to a spherical region of volume 10^{-33} m^3 is of the order of

(a) 10^{-10} J

(b) 10^{-12} J

(c) 10^{-14} J

(d) 10^{-16} J

26. In a Compton scattering process, a photon of wavelength λ is scattered off a charged particle of mass m (initially at rest) by an angle θ. If the final wavelength of the photon is λ', the difference $\lambda - \lambda'$

(a) depends on θ, but not on λ

(b) depends on λ, but not on m

(c) depends on both λ and θ

(d) depends on θ, but not on m

27. The time-dependent wave function of a particle of mass m moving in one dimension under the influence of a potential $V(x)$ is given to be

$$\psi(x, t) = \begin{cases} \alpha x e^{-\beta x} e^{i\gamma t/\hbar} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

where α, β and γ are real numbers. For $x > 0$, the potential $V(x)$ is of the form (k_1 and k_2 are constants)

(a) $k_1 + \frac{k_2}{x}$

(b) $k_1 + \frac{k_2}{x^2}$

(c) $k_1 + k_2 x$

(d) $k_1 + k_2 x^2$

28. Consider an ensemble of quantum particles each of which can be in one of two states of energy E_1 and E_2. This system is in equilibrium at temperature $T = 300$ K. Let N_1 and N_2 denote the average number of particles in the two states. If the ratio N_2 / N_1 is $1/e$, the frequency of the radiation corresponding to transition between the two states is approximately

(a) 62×10^9 Hz

(b) 62×10^{11} Hz

(c) 62×10^{13} Hz

(d) 62×10^{15} Hz

29. A radioactive nucleus X decays to Y with a mean lifetime τ_1. The nucleus Y is also unstable and decays with mean lifetime $\tau_2 (= \tau_1 / 2)$. If N_0 nuclei of X (but no nuclei of Y) are present at $t = 0$, how many nuclei of Y will be there when the number of X nuclei becomes $N_0 / 2$?

(a) $N_0 / 2e$

(b) $N_0 / 4$

(c) $N_0 / 2$

(d) N_0 / e

30. A beam of light moves in a slab of glass of refractive index n in the positive x-direction. The slab itself is also moving in the positive x-direction with a speed v in the laboratory frame. What is the speed of the beam of light as measured in the laboratory frame?

(a) c

(b) $(c^2 n + cv) / (c + nv)$

(c) $c(1 - \frac{1}{n})$

(d) $(c^2 + vcn) / (cn + v)$

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ENTRANCE EXAMINATION, 2010

M.Sc. PHYSICS

[Field of Study Code : SPSM (227)]

Time Allowed : 3 hours

Maximum Marks : 90

INSTRUCTIONS FOR CANDIDATES

(i) All questions are compulsory.

(ii) For each question, one and only one of the four choices given is the correct answer.

(iii) For each question, **answer must be given in the space provided in the SHEET-I (Answer Table)**. Answer given in any other place will not be evaluated.

(iv) Each correct answer will be given +3 marks.

(v) Each wrong answer will be given −1 mark.

(vi) Use of calculators is permitted.

(vii) One extra page is attached at the end of the Question Paper for Rough Work.

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1. The eigenvalues of the matrix $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ are

(a) $+1, +1, +1$ and $+1$

(b) $+1, +1, -1$ and -1

(c) $0, 0, \sqrt{2}$ and $-\sqrt{2}$

(d) $+\sqrt{2}, -\sqrt{2}, +1/\sqrt{2}$ and $-1/\sqrt{2}$

2. Three dices with faces marked 1, 2, ⋯, 6 are thrown together. Assuming that they are unbiased, what is the probability that the sum of the numbers that turn up is 15?

(a) 1/108

(b) 3/108

(c) 5/108

(d) 7/108

3. What is the value of the definite integral $\int_0^1 (x \ln x)^2 dx$?

(a) 1/27

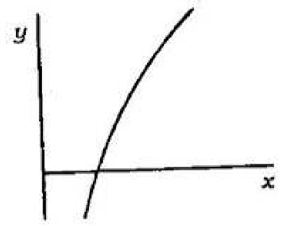
(b) 2/27

(c) 1/9

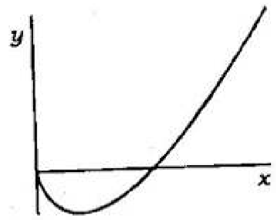
(d) 2/9

4. Which of the following graphs gives the best representation of the real-valued function $y = x \ln x$ in the domain $x > 0$?

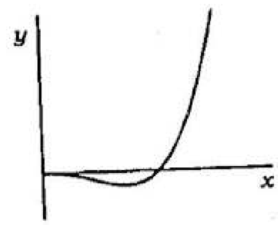
(a)



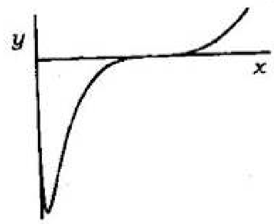
(b)



(c)



(d)



5. A particle of mass m moves in the xy-plane in the presence of a potential $V(x, y)$ so that its Lagrangian is given by

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}(x^2 + y^2) + 2\ln|x - y|$$

Which of the following statements corresponds to the equilibrium of the system?

(a) There is no stable equilibrium at any finite values of (x, y)

(b) There is only one stable equilibrium at the point $(x, y) = (0, 0)$

(c) There are two stable equilibria at the points $(x, y) = (1, 1)$ and $(-1, -1)$

(d) There are two stable equilibria at the points $(x, y) = (1, -1)$ and $(-1, 1)$

6. A uniform spring of spring constant k is cut into two pieces such that one piece is three times as long as the other. The spring constant of the short piece is

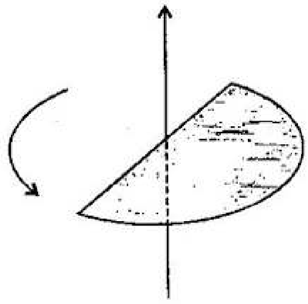
(a) $k/4$

(b) k

(c) $4k/3$

(d) $4k$

7. A half-disc of mass M and radius R rotates about an axis that passes through the centre of the straight side and is perpendicular to its plane (see figure). The moment of inertia about the axis of rotation is



(a) $MR^2/8$

(b) $MR^2/4$

(c) $MR^2/2$

(d) $2MR^2$

8. Consider a pipe of length 65 cm with both ends open. How many modes of oscillations of the air column are possible up to a frequency of 1250 Hz ? (Take the speed of sound to be 330 m/s)

(a) 2

(b) 3

(c) 5

(d) 6

9. A plane wave of frequency ω propagates so that planes of constant phase move with speeds v_1, v_2 and v_3 along the x, y and z axes, respectively. The wave vector, $\mathbf{K}$ of the plane wave (in terms of the Cartesian unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$) is

(a) $\mathbf{K} = \omega\left(\frac{1}{v_1}\hat{i} + \frac{1}{v_2}\hat{j} + \frac{1}{v_3}\hat{k}\right)$

(b) $\mathbf{K} = \omega(\hat{i} + \hat{j} + \hat{k}) / \sqrt{v_1^2 + v_2^2 + v_3^2}$

(c) $\mathbf{K} = \omega(v_1\hat{i} + v_2\hat{j} + v_3\hat{k}) / (v_1^2 + v_2^2 + v_3^2)$

(d) $\mathbf{K} = (v_1\hat{i} + v_2\hat{j} + v_3\hat{k}) / \omega$

10. The concentration $\rho(\mathbf{r}, t)$ of ink diffusing in water is governed by the diffusion equation

$$\frac{\partial}{\partial t}\rho(\mathbf{r}, t) = D\nabla^2\rho(\mathbf{r}, t)$$

where D is a parameter known as the diffusion constant. What is the average time taken for a molecule of ink to spread by a root-mean-square distance R?

(a) $\sqrt{R/D}$

(b) $R/\sqrt{D}$

(c) R^2/D

(d) RD

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11. The ratio F_C / F_G of the electrostatic Coulomb force F_C to the gravitational force F_G between the proton and the electron in the first Bohr orbit (of radius r_B) of a hydrogen atom is closest to the following value

(a) 2×10^{39}

(b) 2×10^{49}

(c) $2 \times 10^{42} r_B$

(d) $-2 \times 10^{35} / r_B$

12. Two point charges q and $4q$ are located at $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, on the xy-plane. The magnitude Q and location $\mathbf{R}$ of a third charge to be placed on the plane such that the total force on each of the three charges vanishes, are

(a) $Q = -4q/9$ and $\mathbf{R} = (2\mathbf{r}_1 + \mathbf{r}_2)/3$

(b) $Q = -4q/5$ and $\mathbf{R} = (\mathbf{r}_1 + 2\mathbf{r}_2)/3$

(c) $Q = -4q/25$ and $\mathbf{R} = (16\mathbf{r}_1 + \mathbf{r}_2)/\sqrt{17}$

(d) $Q = -9q/4$ and $\mathbf{R} = (\mathbf{r}_1 + 2\mathbf{r}_2)/3$

13. Consider a spherical cavity in an infinite, homogeneous and isotropic dielectric material of permittivity ε. When placed in an external electric field $\mathbf{E}$, the electric field inside the cavity is

(a) $3\varepsilon\mathbf{E} / (2\varepsilon + \varepsilon_0)$

(b) $(2\varepsilon + \varepsilon_0)\mathbf{E} / 3\varepsilon$

(c) $-\varepsilon\mathbf{E}$

(d) $\mathbf{E} / \varepsilon$

14. A circular coil of radius 120 mm with 10 turns is placed with its plane parallel to the earth's magnetic field. When a current of 0·45 A flows through the coil, a compass needle placed at the centre of the coil is seen to be deflected by an angle of 45° to the plane of the coil. The intensity of the earth's magnetic field is calculated to be

(a) $0 \cdot 3\ \mu T$

(b) $24\ \mu T$

(c) $42\ \mu T$

(d) $100\ \mu T$

15. Electric current of uniform current density is flowing through an infinitely long wire with a circular cross-section of constant radius R. Which of the following relations gives the magnitude B of the magnetic field measured inside the wire at a distance r from the central axis of the wire (so that $r < R$)?

(a) $B \propto r$

(b) $B \propto \ln r$

(c) $B \propto 1/r$

(d) $B \propto 1/r^2$

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16. An amplifier has a voltage gain $A_V = 1000$, input impedance 1 kΩ and output impedance 500 Ω. A fraction $\beta_V = 0 \cdot 1$ of the output voltage is fed back in series in opposition to the input voltage. The input and output impedance after the feedback are given respectively by the approximate values

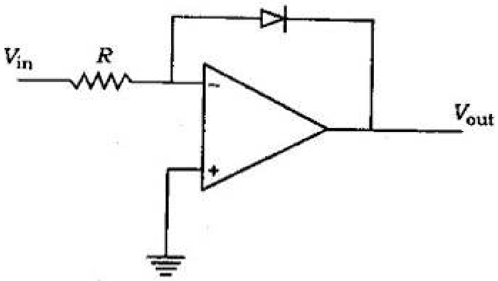
(a) 100 kΩ and 5 Ω

(b) 1 kΩ and 5 Ω

(c) 10 Ω and 50 kΩ

(d) 100 kΩ and 100 Ω

17. In the circuit shown below, the output voltage is proportional to



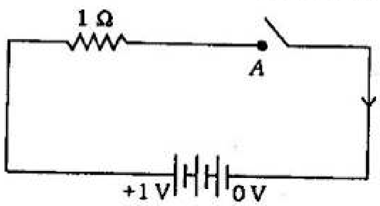
(a) $\ln V_{in}$

(b) $\exp(V_{in})$

(c) $|V_{in}|$

(d) $-|V_{in}|$

18. Which of the following statements is true regarding the voltage V measured at point A and the steady-state value of current I in the circuit shown below?



(a) $I = 0$ A and $V = 0$ V

(b) $I = 0$ A and $V = 1$ V

(c) $I = 1$ A and $V = 1$ V

(d) $I = 0$ A and V fluctuates randomly until the switch is closed

19. An ideal gas undergoes an isothermal expansion (at a constant temperature T) from an initial volume V_1 to a final volume V_2. The change in the entropy per mole is

(a) $R(V_1/V_2)$

(b) $R\ln|V_1-V_2|$

(c) $R\ln(V_1/V_2)$

(d) $R\ln(V_2/V_1)$

20. A gas at a pressure p_A and volume V_A is compressed adiabatically to a volume V_B at pressure p_B. If the pressure-volume relation for this gas during adiabatic compression is $pV^{5/3}=K$ (where K is a constant), the work done during the compression is

(a) $K\left(\frac{1}{V_B^{4/3}}-\frac{1}{V_A^{4/3}}\right)$

(b) $\frac{5}{2}K\left(\frac{1}{V_B^{2/3}}-\frac{1}{V_A^{2/3}}\right)$

(c) $\frac{3}{2}(p_BV_B-p_AV_A)$

(d) $\frac{5}{3}(p_BV_B-p_AV_A)$

21. A gas, the molecules of which have mass m, is at equilibrium at absolute temperature T. The root-mean-square of the *relative velocity* between any two molecules of the gas is

(a) $4k_BT/\pi m$

(b) $3k_BT/2m$

(c) $3k_BT/m$

(d) $6k_BT/m$

22. The energy loss due to diffraction for a plane wave reflected back and forth between two plane mirrors

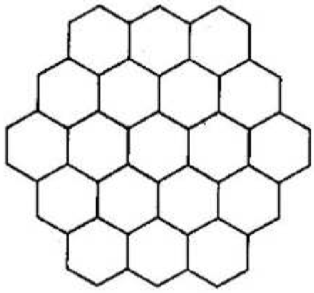
(a) decreases with an increase in the size of the mirrors

(b) increases with an increase in the size of the mirrors

(c) decreases with an increase in the separation between the mirrors

(d) increases with a decrease in the wavelength of the light

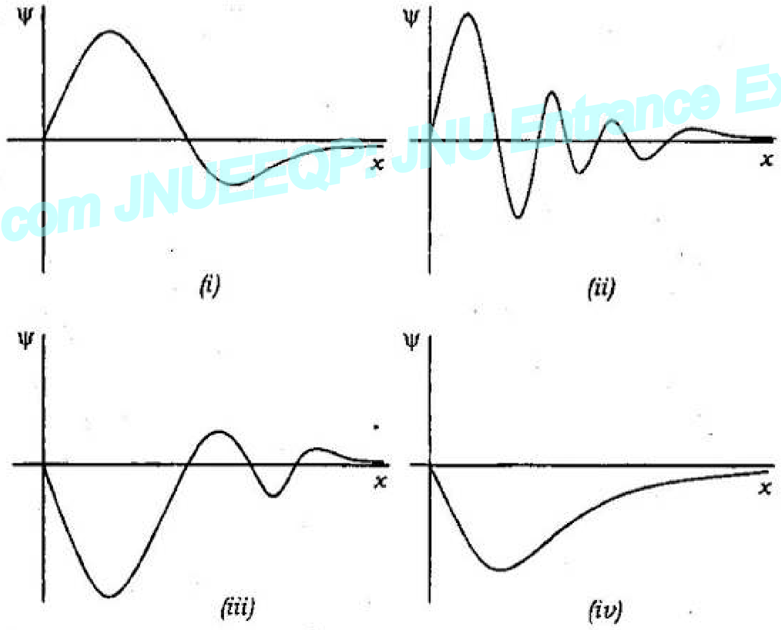
23. In a single layer of graphite, called graphene, the carbon atoms form a hexagonal lattice (see figure). How many carbon atoms are there in a unit cell of the lattice?



(a) One (b) Two

(c) Three (d) Six

24. A quantum particle in one dimension x moves under the influence of a potential that supports bound states. Plotted below are the wave functions ψ vs. x corresponding to four eigenstates of energy. Identify the wave functions corresponding to the ground state and the highest energy state among those shown :



(a) *(i)* corresponds to the ground state and *(iii)* to the highest energy state

(b) *(ii)* corresponds to the ground state and *(iv)* to the highest energy state

(c) *(iv)* corresponds to the ground state and *(ii)* to the highest energy state

(d) None of these is the ground state and *(ii)* corresponds to the highest energy state

25. An X-ray machine operates at a potential of 50000 V. What is the minimum possible value of wavelength present in the radiation?

(a) 0·0124 nm

(b) 0·0248 nm

(c) 0·124 nm

(d) 0·248 nm

26. The eigenstates of $\hat{S}_z$ of a quantum particle with spin ½ are $|\uparrow\rangle = \left|s = \frac{1}{2}, s_z = \frac{1}{2}\right\rangle$ and $|\downarrow\rangle = \left|s = \frac{1}{2}, s_z = -\frac{1}{2}\right\rangle$. Which of the following statements about eigenstates of the operator $\hat{S}_x$ is true?

(a) $|\uparrow\rangle$ is an eigenstate of $\hat{S}_x$

(b) $|\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(c) $|\uparrow\rangle + |\downarrow\rangle$ is an eigenstate of $\hat{S}_x$

(d) No linear combination of $|\uparrow\rangle$ and $|\downarrow\rangle$ can be an eigenstate of $\hat{S}_x$

27. Consider a quantum particle of mass m in a three-dimensional isotropic simple harmonic potential $V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$. It is known that the particle is in an energy eigenstate with eigenvalue $E = 7\hbar\omega/2$. Which of the following *cannot* be the wave function of the particle? (In the following $\alpha = \sqrt{m\omega/\hbar}$ and $H_n(\xi)$ is the nth Hermite polynomial)

(a) $H_2(\alpha x)\exp(-\alpha(y^2 + z^2))$

(b) $H_2(\alpha x)\exp(-\alpha(x^2 + y^2 + z^2))$

(c) $H_1(\alpha y)H_1(\alpha z)\exp(-\alpha(x^2 + y^2 + z^2))$

(d) $H_1(\alpha x)H_1(\alpha z)\exp(-\alpha(x^2 + y^2 + z^2))$

28. In order to determine the age of ancient wooden tools, radiocarbon dating method is used. This is done by measuring the fraction of radioactive isotope ^{14}C of carbon compared to the normal (non-radioactive) isotope ^{12}C in a sample. An old sample is found to contain 1/10 times the fraction of ^{14}C as compared to a fresh piece of wood. Given that the half-life of ^{14}C is 5570 years, the approximate age of the old sample is

(a) 557 years

(b) 12800 years

(c) 18500 years

(d) 55700 years

29. In the Large Hadron Collider (LHC) in CERN, protons will be accelerated to an energy of 7 TeV (tetraelectron volt), i.e., 7×10^{12} eV: (As a matter of fact, two proton beams, each with 7 TeV will collide so that the energy of collision in the centre of mass frame is 14 TeV.) The speed υ of a proton of 7 TeV energy is in the range

(a) $0 \cdot 9999990000c < \upsilon < 0 \cdot 9999999999c$

(b) $0 \cdot 999900c < \upsilon < 0 \cdot 999999c$

(c) $0 \cdot 9900c < \upsilon < 0 \cdot 9999c$

(d) $0 \cdot 90c < \upsilon < 0 \cdot 99c$

30. In the special theory of relativity, consider a Lorentz boost by a velocity υ along the x-direction. If $u = ct + x$, then the boosted value $u' = ct' + x'$ is

(a) $u' = \sqrt{\frac{1+\upsilon/c}{1-\upsilon/c}}u$

(b) $u' = \sqrt{\frac{1-\upsilon/c}{1+\upsilon/c}}u$

(c) $u' = \frac{1+\upsilon/c}{1-\upsilon/c}u$

(d) $u' = \frac{1-\upsilon/c}{1+\upsilon/c}u$

★★★

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ENTRANCE EXAMINATION, 2009

M.Sc. PHYSICS

[Field of Study Code : SPSM (216)]

Time Allowed : 3 hours

Maximum Marks : 90

INSTRUCTIONS FOR CANDIDATES

(i) All questions are compulsory.

(ii) For each question, one and only one of the four choices given is the correct answer.

(iii) For each question, **answer must be given in the space provided in the SHEET–I (Answer-Table)**. Answer given in any other place will not be evaluated.

(iv) Each correct answer will be given +3 marks.

(v) Each wrong answer will be given −1 mark.

(vi) Use of calculators is permitted.

(vii) Extra pages are attached at the end of the Question Paper for Rough Work.

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1. The sum of the infinite series

$$1-\frac{\pi^2}{2!\,2^4}+\frac{\pi^4}{4!\,2^8}-\frac{\pi^6}{6!\,2^{12}}+\cdots$$

is

(a) convergent and equals zero

(b) convergent and equals $1/\sqrt{2}$

(c) convergent and equals $\sqrt{2}$

(d) divergent

2. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ are

(a) -1, -1 and 2

(b) -1, 1 and 2

(c) -1, -2 and 2

(d) -1, 2 and 2

3. Planck's constant (h), the speed of light in vacuum (c) and Newton's gravitational constant (G) are three fundamental constants of nature. Which of the following combinations has the dimension of length?

(a) $\sqrt{hG}/c^{3/2}$

(b) $\sqrt{hG}/c^{5/2}$

(c) $\sqrt{hc/G}$

(d) $\sqrt{Gc}/h^{3/2}$

4. Consider a simple harmonic oscillator made up of a mass attached to a spring. What will be the effect on the motion if the mass were to stay completely immersed in a liquid?

(a) The motion is simple harmonic but with a frequency which is lower compared to when it is in air

(b) The motion is always of the damped harmonic type

(c) The motion is always overdamped

(d) The motion is either of the damped harmonic type, or it is overdamped, depending on the liquid

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5. The kinetic energy T and the potential energy V of a spring-mass system executing a simple harmonic motion vary with time. At an instant of time, at which the product TV assumes its maximum value, let the displacement be a fraction f of the amplitude of oscillation. The absolute value of f is

(a) $1/\sqrt{3}$

(b) $1/\sqrt{2}$

(c) $1/3$

(d) $1/2$

6. Three identical rings of radius a are placed on the xy-plane so that each ring touches the other two tangentially. Each ring, which may be considered to be infinitesimally thin, has a mass M. The centres of the three rings are equidistant from the z-axis—this common distance being $2a/\sqrt{3}$. What is the moment of inertia of this system about the z-axis?

(a) $3Ma^2/4$

(b) $3Ma^2/2$

(c) $6Ma^2$

(d) $7Ma^2$

7. A particle of mass 1 kg is moving along the x-axis. There is no force on the particle except during the time interval between $t = 0$ s and $t = 5$ s, when a force of the form $F(t) = 12t^2$ N/s^2 acts on it. If the velocity of the particle at $t = 0$ is 500 m/s, its velocity at $t = 10$ s is

(a) 800 m/s

(b) 1000 m/s

(c) 1700 m/s

(d) 4500 m/s

8. Raindrops hit the ground with a terminal velocity that is achieved due to a balance between the gravitational force and viscous drag force of air. Assume a raindrop to be a perfectly spherical water droplet of radius R. The momentum that it transfers when it hits the ground is proportional to

(a) R

(b) R^2

(c) R^3

(d) R^5

9. A wooden block B is at rest on the horizontal surface of a wooden platform P. The coefficient of static friction between these two is μ. The platform starts moving suddenly with a uniform acceleration a. The minimum value of a so that the block B starts moving with respect to the platform P is (g denotes the acceleration due to gravity)

(a) $(1-\mu)g$

(b) $(1+\mu)g$

(c) μg

(d) $(1-\mu^2)g$

10. An ideal gas expands in a process in which its pressure depends on the volume as $p = p_0 e^{-\alpha V}$, where p_0 and α are two positive constants. If n is the number of moles in the gas, the maximum temperature that it will attain in this process is (R is the universal gas constant)

(a) $p_0 / e\alpha nR$

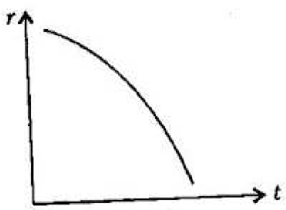
(b) $p_0 / \alpha n^3 R$

(c) $e p_0 / \alpha nR$

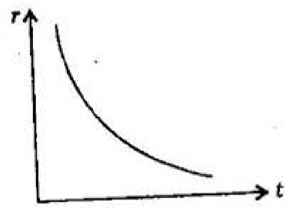
(d) $p_0^2 / \alpha nR$

11. Consider the process of melting of a spherical ball of ice. Assume that the heat is being absorbed uniformly through the surface and that the rate of absorption is proportional to the instantaneous surface area. Which of the following graphs depicts the change of radius r (vertical axis) as a function of time t (horizontal axis)?

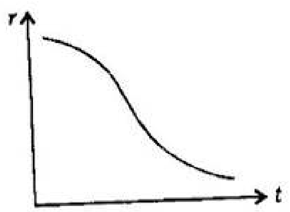
(a)



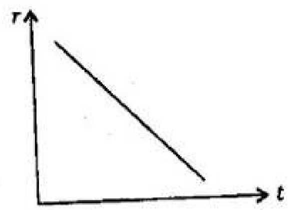
(b)



(c)



(d)



12. An ideal gas undergoes an isothermal change of volume. The initial and final volumes are given to be 1·0 litre and 2·7 litres respectively. If in this process the entropy per mole changes from S_1 to S_2, the value of the difference $S_2 - S_1$ is nearest to (R is the universal gas constant)

(a) $+2\cdot7R$

(b) $+1\cdot7R$

(c) $+1\cdot0R$

(d) $-1\cdot0R$

13. Let U, T, S and P denote, respectively, the internal energy, temperature, entropy and pressure of a thermodynamic system. Then a change ΔF in the free energy $F = U - TS$ measures

(a) the heat exchanged at constant pressure

(b) the work done by the system at constant entropy

(c) the work done on the system at constant temperature

(d) the heat exchanged at constant temperature

14. Which of the following integrals depends only on the initial and final states of a thermodynamic system (i.e., independent of the path of transformation)?

(a) $\int p dV$

(b) $\int dQ$

(c) $\int \frac{dQ}{T}$

(d) $\int T^2 dS$

15. Two cylindrical rods A and B of identical length and cross-section are joined end-to-end. The free ends of A and B are at 20 °C and 40 °C respectively. The thermal conductivity of A is three times that of B. In the steady state, the temperature at the junction of A and B is

(a) 25 °C

(b) 30 °C

(c) 35 °C

(d) $\sqrt{800}$ °C

16. The amount of energy required to charge a metallic sphere of radius 2·0 m to a potential of 3000 V in air is

(a) 1×10^{-3} joule

(b) 2×10^{-3} joule

(c) 3×10^{-3} joule

(d) 4×10^{-3} joule

17. In a region of space that does not contain any electric charge, the electrostatic potential satisfies Laplace's equation. Suppose that the potential in such a situation is given by the formula

$$V(x, y, z) = A\left(bx^2 + \frac{1}{2}y^2 - z^2\right)$$

where A and b are constants. Then the value of b

(a) can be arbitrary

(b) must be zero

(c) must be 1/2

(d) must be 1

18. Suppose that the electric field on the xy-plane is given by

$$\vec{E} = P\left[xy\hat{i} + \left(\frac{1}{2}x^2 + y^2\right)\hat{j}\right]$$

The magnitude of the difference in potential between the origin (0, 0) and the point (1, 2) is

(a) $\frac{5}{4}P$

(b) $\frac{11}{3}P$

(c) $\frac{7}{2}P$

(d) not unique, it depends on the path joining the two points

19. Four electric charges of strength Q, Q, $-Q$ and $-Q$ are placed respectively at the points $(-b, 0)$, $(b, 0)$, $(0, -a)$ and $(0, a)$ on the xy-plane. At a distance d very far away from these charges ($d >> a, b$), the electric field will appear to be approximately that of

(a) a point charge

(b) a dipole

(c) a quadrupole

(d) a mixture of a dipole and a quadrupole

20. The potential due to an electric dipole $\mathbf{p}$ placed at the origin is known to be of the form $\phi(\mathbf{r}) = \mathbf{p} \cdot \mathbf{r} / r^3$. The total flux of the electric field through a spherical surface of radius R with the dipole at the centre is given by

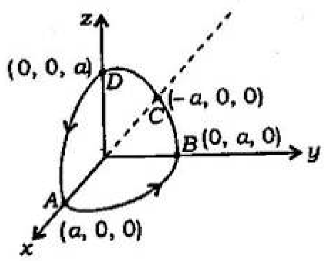
(a) zero

(b) $-\frac{2\pi}{3}\frac{|\mathbf{p}|}{R}$

(c) $+\frac{2\pi}{3}\frac{|\mathbf{p}|}{R}$

(d) $+\frac{4\pi}{3}\frac{|\mathbf{p}|}{R}$

21. In the following figure, ABC and CDA are two semicircular conducting wires of radius a with the origin as their common centre. The arc ABC lies on the xy-plane whereas CDA lies on the xz-plane. They are joined as shown. This combined loop carries a current in the direction indicated in the figure below. Which of the following unit vectors denotes the direction of the resulting magnetic field vector at the origin?



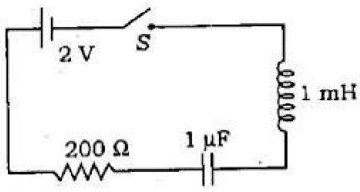
(a) $(1, 0, 0)$

(b) $\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

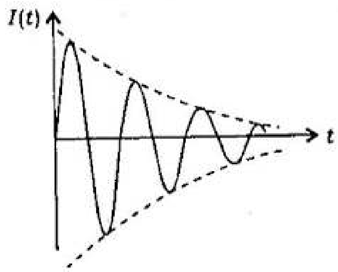
(c) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

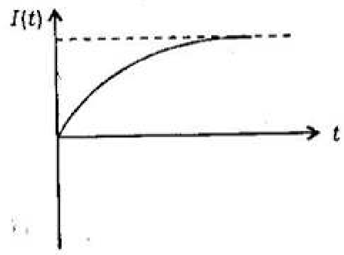
(d) $(-1, 0, 0)$

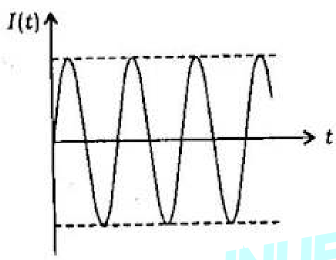
22. In the LCR circuit shown below, the switch S is suddenly closed at $t = 0$:

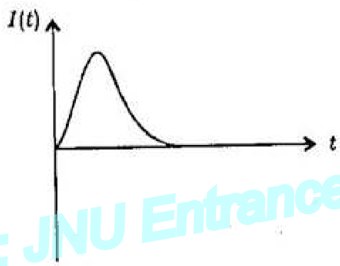


Which of the following diagrams represents the subsequent flow of current $I(t)$ as a function of time t?

(a)


(b)


(c)


(d)


23. The kinetic energy of a free relativistic particle is $E - mc^2$, where E and m are its total energy and rest mass respectively. Let v_0 be the speed at which the kinetic energy equals the rest mass energy of the particle. Then

(a) $v_0 = c/\sqrt{2}$

(b) $v_0 = \sqrt{3}c/2$

(c) $v_0 = c$

(d) $v_0 > c$ (so this can never happen)

24. A galaxy is receding relative to us at a speed of 3000 km/s. It emits hydrogen redline of wavelength 6560 Å. When seen by us, the wavelength of this radiation will appear to be

(a) higher by approximately 65 Å

(b) lower by approximately 65 Å

(c) lower by approximately 6 Å

(d) higher by approximately 6 Å

25. The angular frequency ω of deep water waves varies as the inverse square root of the wavelength λ, i.e., $\omega(\lambda) \propto 1/\sqrt{\lambda}$. Which of the following is the relation between its group velocity υ_g and phase velocity υ_p?

(a) $\upsilon_g = \upsilon_p/2$

(b) $\upsilon_g = \upsilon_p$

(c) $\upsilon_g = 2\upsilon_p$

(d) $\upsilon_g = \lambda\upsilon_p$

26. Suppose you are using light of wavelength 6250 Å in combination with a plane diffraction grating which has 1600 lines per cm. The upper limit on the order of diffraction maxima that can possibly be seen with this arrangement is closest to

(a) 25

(b) 18

(c) 10

(d) 2

27. In a Young's double-slit experiment, the two slits are such that the dark fringes are perfectly dark (i.e., the intensity vanishes at these points). Now, we double the width of one of the two slits. What will be the effect on the interference pattern?

(a) The fringe pattern as well as the contrast between the dark and bright fringes will change

(b) The fringe pattern will not change but the contrast between the dark and bright fringes will change

(c) The fringe pattern will change but the contrast between the dark and bright fringes will not change

(d) There will be no change in either the interference pattern or the contrast between the dark and bright fringes

28. The wavelength of one of the visible lines in the spectrum of the hydrogen atom is approximately 6560 Å. It is known that this corresponds to a transition between the states with the principal quantum numbers $n = 2$ and $n = 3$. What will be the frequency corresponding to the transition from the ground state to the first excited state?

(a) $4 \cdot 56 \times 10^{14}$ Hz

(b) $1 \cdot 65 \times 10^{15}$ Hz

(c) $2 \cdot 47 \times 10^{15}$ Hz

(d) $3 \cdot 29 \times 10^{15}$ Hz

29. The intensity *vs.* wavelength distribution of a blackbody is found to have its maximum at the wavelength $\lambda_{max} = 0 \cdot 2$ cm. Moreover, the total energy radiated J is measured to be 30 watts. If the temperature of the blackbody is now doubled, the new values of λ_{max} and J will be

(a) 0·1 cm and 120 watts

(b) 0·1 cm and 240 watts

(c) 0·1 cm and 480 watts

(d) 0·4 cm and 60 watts

30. The radioactivity of a sample of Co^{55} decreases by 4% every hour. (The decay product is not radioactive.) The half-life of Co^{55} is approximately

(a) 1 hour

(b) 1 day

(c) 1 month

(d) 1 year

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ENTRANCE EXAMINATION, 2008

M.Sc. PHYSICS

Time Allowed : 3 hours

Maximum Marks : 100

INSTRUCTIONS FOR CANDIDATES

(i) The question paper consists of two parts—Part A and Part B.

(ii) All questions are compulsory. Answers should be written in the space following each question.

(iii) Use of calculators is permitted.

(iv) Extra pages are attached at the end of the question paper for Rough Work.

PART—A

Note : Answer **all** questions. Each question carries 10 marks.

A1. Show that

$$\sum_{n=1}^{\infty} \frac{\sin(nx)}{n} = \frac{(\pi - x)}{2}, \quad 0 < x \leq \pi$$
$$= -\frac{(\pi + x)}{2}, \quad -\pi \leq x < 0$$

A2. A harmonic oscillator consists of a mass m attached to a spring with spring constant k. It is immersed in a fluid which damps the motion with friction coefficient b.

(a) Write down the differential equation which describes the time-dependence of the displacement $x(t)$ of the mass from its equilibrium position.

(b) Solve this equation, and identify parameter ranges for which the motion is damped oscillatory or only damped.

A3. The free energy of a cluster with N molecules has contributions from both volume and surface terms as follows :

$$\Delta G = \Delta_0 V + \gamma S$$

Here Δ_0 is the free-energy cost (per unit volume) of bringing the particles together, γ is the surface tension, and V, S denote the volume and surface area of the cluster, respectively. Assume that the cluster is spherical in shape and each molecule has a volume v_m so that $V = Nv_m$.

(a) Express ΔG in terms of N.

(b) Assuming $\Delta_0 < 0$, obtain the value of N for which ΔG is maximum. Sketch ΔG as a function of N.

(c) Compute the radius of the cluster in *(b)*.

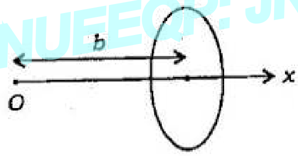
A4. A magnetic dipole of moment μ is placed along the x-axis at the origin.

(a) Write down the magnetostatic potential, and the value of the magnetic field at an arbitrary point $\vec{r}$. (No derivation is required.)

(b) The dipole executes oscillations along the x-axis with a small amplitude A and angular frequency ω, so that the position in time is

$$x(t) = A\cos(\omega t)$$

A coil of small area S is placed perpendicular to the x-axis at a distance b as shown. Calculate the e.m.f. induced in the coil. (Neglect the variation of the field across the coil and take $A << b$.)



A5. "A beam of thermal neutrons falling on a crystal gives rise to diffraction phenomena analogous to those observed with X-rays." Justify this statement by estimating de Broglie wavelength of a neutron with a speed corresponding to the average thermal energy at 300 K.

PART—B

Note : Answer **all** questions. Each question carries 5 marks.

B1. Compute the eigenvalues and normalized eigenvectors of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

B2. A factory has four workers A, B, C and D who produce 1%, 3%, 5% and 7% defective items, respectively. Furthermore, A, B, C and D work for 40%, 30%, 20% and 10% of the time, respectively. If a defective item is produced, what is the probability that it was produced by A ?

32/65

B3. A ball hits the ground at an angle of 45° with a speed of 1 m/s. After every bounce, the vertical component of velocity is reduced in magnitude by 10 percent. What horizontal distance does the ball travel between the first bounce and the tenth bounce?

B4. Write down the expression for Maxwell's velocity distribution law and speed distribution law. Find the most probable value of the particle speed. Obtain its ratio with the root-mean-square speed of the particles.

B5. Prove the thermodynamic relation

$$\left.\frac{\partial \mu}{\partial N}\right|_{V,T} = -\frac{V^2}{N^2}\left.\frac{\partial P}{\partial V}\right|_{N,T}$$

where the symbols have their usual meanings.

B6. A particle of charge q is placed near a large planar metallic sheet. The perpendicular distance of the charge from the sheet is d. Calculate the force of attraction exerted by the sheet on the charge.

B7. A proton is moving with a high speed so that its mass exceeds the neutron rest mass. Under these conditions, can it undergo the following decay?

$$p \rightarrow n + e^{+} + \nu_e$$

B8. A sample containing hydrogen atoms is subjected to an external oscillating magnetic field of frequency 34 MHz. When another constant magnetic field of magnitude 0·78 T is applied, proton spin-flipping is observed. Calculate the strength of the local magnetic field (due to the magnetic moments of the atoms and nearby nuclei) at the site of the protons that are undergoing spin flips, assuming the external and local fields are parallel there. The protons have a dipole moment $\mu = 1{\cdot}41 \times 10^{-26}$ J/T.

B9. A particle of mass m moves in a one-dimensional potential $V(x) = |x|$. Using the Bohr-Sommerfeld quantization condition, or otherwise, find the n-dependence of the nth energy level E_n.

B10. The compound NaCl has a cubic lattice with density 2·167 g/cm^3. What is the distance between adjacent atoms? Na has a molar mass of 23 g/mol, and Cl has a molar mass of 35·4 g/mol.

65

Total Pages : 21

ENTRANCE EXAMINATION, 2007

M.Sc. PHYSICS

Time Allowed : 3 hours

Maximum Marks : 100

INSTRUCTIONS FOR CANDIDATES

(i) The question paper consists of two parts—Part A and Part B.

(ii) All questions are compulsory. Answers should be written in the space following each question.

(iii) Use of simple calculators is permitted.

(iv) Extra pages are attached at the end of the question paper for Rough Work.

PART—A

Note : Answer **all** questions. Each question carries 10 marks.

A1. Consider a particle of mass m moving along the x-axis in a potential of the form $V(x) = x^3 - 3x + 3$.

(a) Draw a schematic graph of this potential.

(b) Are there values of x at which the particle can be stationary in equilibrium? Indicate the nature of the equilibrium (stable or unstable) in each case. If there is a point of stable equilibrium, calculate the frequency of oscillation around it (for small enough amplitude).

A2. A particle of mass m moves in a one-dimensional box located between $x = 0$ and $x = L$. The wave function is given by $\psi(x) = Cx(L - x)$, where C is the constant of normalization. Calculate the expectation value of the kinetic energy of the particle.

3. Let $f(=F/V)$ be the Helmholtz free energy per unit volume for a gas of N particles.

(a) Show that the equilibrium pressure P is obtained as

$$P = n\left(\frac{\partial f}{\partial n}\right)_T - f$$

where $n(=N/V)$ is the number of particles per unit volume, and T is the temperature of the gas.

(b) For an ideal gas, show that the free energy per particle $\tilde{f}\ (=F/N)$ is obtained as $\tilde{f} = k_B T \ln(n) + C_T$, where C_T is a temperature-dependent constant.

A4. Consider a random walk of a person along a straight line. Each step is of length l and is equally likely to be in the forward or in the backward direction. After n such steps are taken

(a) prove that the probability of the net displacement from the starting point being equal to rl in the forward direction is given by

$$P(r) = \frac{n!}{[(n-r)/2]![(n+r)/2]!}\left(\frac{1}{2^n}\right)$$

(b) show that the average value of r is equal to 0;

(c) show that the average value of r^2 is equal to n.

A5. Using Bohr's theory of an electron in a hydrogen atom and the classical theory of electromagnetism, calculate the strength of the magnetic field at the location of the nucleus due to electronic motion. Assume that the electron is in the ground state.

PART—B

Note : Answer **all** questions. Each question carries 5 marks.

B1. An L-C oscillator consists of an inductor with inductance L and a capacitor of capacitance C. A sinusoidal current with amplitude I_0 is flowing through the circuit. Calculate the frequency of the current and the average energy in the capacitor.

B2. Solve the ordinary differential equation

$$x\frac{du}{dx} - (x+1)u = 0$$

given that $u(x=1) = 1$.

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B3. Estimate the maximum distance at which the eye is able to resolve the two headlights of an approaching car, given the following information : *(a)* Distance between the headlights is 1·4 m, *(b)* the pupil diameter of the observer's eye is 5 mm and *(c)* the wavelength of the light is 550 nm. Assume that the diffraction effects alone limit the resolution.

B4. A muon is an elementary particle of rest mass energy equal to 105 MeV. It decays with a mean lifetime of $2{\cdot}2 \times 10^{-6}$ seconds in its rest frame. Consider muons which are being produced with a kinetic energy of 70 MeV. What is the average distance that such a muon travels before it decays?

B5. An electric charge Q is placed at the point $(x = y = 0, z = d)$. Calculate the electric field flux through the circle $x^2 + y^2 = a^2$ in the $z = 0$ plane.

B6. Find the average velocity of the free electrons producing a current of 5 amperes in a copper wire having a cross-sectional area of 4·0 square mm. Assume that each copper atom contributes one mobile electron. Density of copper is 9×10^3 kg/m^3. Atomic weight of copper is 63·5 a.m.u.

B7. Suppose you are computing the derivative of a function f at the point x by evaluating the function numerically at two different points separated by h. For a given value of h which of the following two expressions constitutes a more accurate representation of the derivative?

(a) $[f(x + h/2) - f(x - h/2)]/h$

(b) $[f(x + h) - f(x)]/h$

Explain your answer.

B8. Suppose you are putting identical hard spheres of diameter σ on a face-centered cubic lattice. In the closest packed configuration of the spheres, calculate the lattice constant of the cubic unit cell in terms of σ.

B9. A reversible cyclic engine operates between two identical bodies of constant (temperature independent) heat capacity. Initially these two bodies are at temperatures T_1 and T_2. The engine comes to rest when these two bodies attain a common temperature T_F. Express T_F in terms of T_1 and T_2.

B10. For a particle of mass m moving in a spherically symmetric potential $V(r)$, it is found that the value of the kinetic energy is twice the value of the potential energy for all circular orbits. Derive the form of $V(r)$.