CE8-R3: LOGIC AND FUNCTIONAL PROGRAMMING

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Convert the following FOL formula into its equivalent Prenex form
$\alpha: (\forall x)\{(\sim R(x) \rightarrow P(a)) \wedge (\exists z) \sim Q(z,a)\} \wedge (\forall x) \{P(x) \rightarrow (\exists y) Q(y,x))\}$

b) Let S = {P(x), Q(x, f(x)) V ~ P(x), ~ Q (g(y), z)} bet set of clauses. Find unsatisfiable set S' of ground instances of clauses in S.

c) Prove a theorem **"from {P V Q, ~ P} infer Q"** using Natural Deduction System.

d) Will the output of the following queries same? If not, give the reason and give a situation when both the queries give same output.

```
? member (X, [2,3,4]).
? not (not (member(X, [2,3,4]))).
```

e) Write λ-abstraction, which generates λ-function for computing successor or predecessor of a number when applied on 1 or –1 respectively.

f) Find normal form of the λ-expression **or true y** and show the evaluation steps.

g) Give recursive datatype definition of a list in SML.

(7x4)

2.

a) Code the following facts and rules as logic program and generate proof tree for the query **'notebook supports pen'**
   i) If X is on top of Y, Y supports X.
   ii) If X is above Y and they are touching each other, X is on top of Y.
   iii) A pen is above a notebook.
   iv) A pen is touching a notebook.

b) Prove {P} |-(~ Q → ~ (P → Q)) using Axiomatic system.

c) Show that the FOL formula ∀y ∃x P(x, y) → ∃x ∀y P(x, y) is satisfiable but not valid.

(7+6+5)

3.

a) Consider the following Prolog program:

```
one(X, Y) :- X > 1, Y > 1.
two(0, 1). two(2, 3).
two(X, Y) :- three(P, Q), three(Q, P), X is P + 1, Y is Q + 1.
four(0). three(1, 3). three(2, 4). three(4, 2).
```

   and the query ?-two(X, Y), not(four(X)), one(X, Y).
   i) What will the output of the following query? Draw the search tree.
   ii) If rule for 'one' in (a) is replaced by one(X, Y) :- X > 1, Y > 1, ! then what will be the output for above query?

b) The following program finds the sum of first N natural numbers.

```
sum(1, 1).
sum(N, R) :- N1 is N – 1, sum(N1, R1), R is R1 + N.
```

   i) Draw search tree for the query? - sum(4, R).
   ii) Is this program terminating?
   iii) If answer to ii) is no, then make it terminating program if possible.
   iv) Write iterative version of this program.

([5+3]+[3+1+3+3])

4.

a) Suppose that a node of a binary tree is coded in Prolog as 'tree(left, data, right)'. Write a predicate called "count(T, N)", which succeeds by binding N with number of leaf nodes in a binary tree T. Empty tree is represented by 'null'.

b) Write a recursive and iterative Prolog programs named 'sum(L, P, N)', where L is list of integers, P is the list of positive integers and N is the list of negative integers in L.

c) Identify what does the following program do?

```
find([], []).
find([H|T],N)          :-   member(H, T), !, find(T, N).
find([H|T],[H|N|)      :-   find(T, N).
```

Give the search tree of the query **?- find([1,2,3,2,3], N).**

(6+6+6)

5.

a) Find normal form of the following λ-expressions and show the evaluation steps.
   i) (λx. x4)(λy. y+6)
   ii) (λx. (λy. x*y)) 7
   iii) (λxy. x*y) 2 5
   iv) (λxy. x y)(λz. z+5) 6

b) Write the expression (x * 3) * 2 – (x * 3)/5 + ((x * 3) + 4) using λ-notation.

c) What do you understand by a normal form of λ-expressions? It is not necessary that all λ-expressions have the normal forms. Justify this statement through example of λ-expression, which does not have normal form.

d) Write a λ-function named 'union' to generate one list by eliminating common elements of two lists, assuming that individual lists have no duplication. Use functions such as null, head, tail, cons(constructor) etc. with the obvious meanings for manipulating lists.

(6+2+5+5)

6.

a) Consider the following function declaration in SML.

```
fun   compute(0, x) = x
      | compute(x, y) = compute (x-1, x*y);
```

   i) Determine the type of function 'compute'.
   ii) Under what condition 'compute' be terminating function?
   iii) Write the evaluation steps for **compute (3, 4)?**

b) Given the following Datatype declaration in SML, write a function to test whether a given day is working or not.

```
datatype DAY = Mon | Tue | Wed | Thr | Fri | Sat | Sun;
```

c) Given datatype for a binary tree. Write a SML function to swap a binary tree at all levels.

(6+6+6)

7.

a) SML does not support array representation. Suggest a scheme for array representation in SML so that any index value of an array can be retrieved in at most ($\log_2 N$) time, where 'N' is number of elements in an array.

b) What are lazy and eager evaluation strategies? Which strategy is used in SML? List two advantages of each.

c) Explain 'let' expression and 'local' declaration in SML. Give a suitable example.

d) How are exceptions declared and handled in SML?

e) How do we achieve polymorphism in SML?

(4+3+5+3+3)