CE8-R3: LOGIC AND FUNCTIONAL PROGRAMMING

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Prove a theorem "from {~P V Q, P} infer Q" using Natural Deduction System.

b) Show that a set containing propositional formulae $\{P, Q, P \rightarrow R, R \rightarrow Q\}$ is inconsistent using truth table.

c) Convert the following FOL formula into its equivalent Prenex form.

$$(\forall y)\ [\ R(y, b)\ \vee (\forall x)\ \{\ Q(x) \rightarrow R(x, y)\ \}]\ \wedge (\forall x)\ (\sim P(x) \rightarrow Q(a)\ )$$

d) State the control strategies of PROLOG.

e) Differentiate between lazy and eager evaluation strategies.

f) Give recursive data type definition of a binary tree in SML.

g) In SML, fn x => E1 | y => E2 is a definition of unnamed function of type 'a --> 'b, where x & y are of type 'a and E1 & E2 are expressions of type 'b. This is similar to lambda functions. Define a construct "if E then E1 else E2" using this notation.

(7x4)

2.

a) Consider $\Sigma = \{ P \vee Q \rightarrow R \wedge Q,\ P \vee Q\}$ a set of propositional expressions. Show that $R \wedge Q$ is logical consequence of Σ using semantic tableaux method.

b) Show that joint denial (neither α nor β) represented $\alpha \downarrow \beta$ is adequate.

c) Prove that, if Σ is a set of hypotheses and α & β are well formed formulae, then prove that $\{ \Sigma \cup \alpha\} \vdash \beta$ implies $\Sigma \vdash (\alpha \rightarrow \beta)$.

(6+6+6)

3.

a) Show that "John has got money" can be concluded from the text given below using resolution refutation method,
"Everyone who sees a movie in a theatre has to buy a ticket. Person who does not have money can not buy a ticket. John sees a movie."

b) Consider the following set of formulae in FOPL:

α : $(\forall x)\ (P(x) \rightarrow (Q(x)\ \wedge R(x)))$
β : $(\exists x)\ (P(x)\ \wedge L(x))$

Show that $G = (\exists x)\ (L(x)\ \wedge R(x))$ is a logical consequence of α and β.

c) Prove that sf S is a set of clauses and then C is a logical consequence of S iff the set $S \cup \{\sim C\}$ is unsatisfiable.

(6+6+6)

4.

a) Code the following facts and rules in prolog and generate search tree for the query 'which courses does Mary take?'. Facts are given as follows:

i) Database is an easy course and AI & Hardware course are not easy.
ii) Books for Hardware and Database courses are available.
iii) AI has 8 credits with no lab component.

Rule 1: X takes Y, if Y is easy course and books for Y are available.

Rule 2: X takes Y, if Y has 8 credits and Y has lab component.

b) Write Prolog program to generate integer number between two bounds (inclusive) say L and U.

(10+8)

5.

a) What will be the values of x and y if the following SML statements are executed?

```
val  pi  =  3.1414;
fun  circum  ( r ) = 2.0 * pi * r;
val x = circum (3.0);
val pi = 1.0;
val y = circum (3.0);
```

b) What are the significances of the following two definitions of averaging two numbers? State at least three differences.

```
fun  av (x,  y) =  (x + y) / 2.0;
fun  av1 x  y  = (x + y) / 2.0;
```

c) Convert the following if-then-else expression into case expression in SML if x = 0 then “zero” else if x = 1 then “one” else if x = 2 then “two” else “none”.

d) Define Boolean implication function denoted by symbol → as an infix operator.

(4+4+5+5)

6.

a) Define a data type SHAPE of geometrical figures square, rectangle and circle. Write a polymorphic function in SML to calculate area of an object of type SHAPE.

b) Write a SML function to merge two integer lists in increasing order assuming original lists are also in increasing order.

c) Write a function in SML to generate a list of 10 numbers of a sequence in increasing order using the following formula:

Number = $2^n * 3^m$, $\forall$ n, m ≥ 0

The list of numbers can be defined informally as:

- 1 is valid number,
- if x is a valid number, then 2x and 3x are also valid numbers.

(6+6+6)

7.

a) Find normal form of the following λ- expressions and show the evaluation steps.

i) (λx . (λy . x * y)) 10

ii) (λx . x 9) (λy . y + 4)

iii) (λxy . x y) (λz . z * 4) 5

iv) (λxy . x * y) 2

b) Define normal form of λ-expressions? Do all λ-expressions have the normal forms? Give arguments to support your answer.

c) Write a λ-function named ‘intersection’ to generate a list common element of two lists. Use functions such as null, head, tail, cons (constructor) etc. with the obvious meanings for manipulating lists.

(6+6+6)