

SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.Tech - IT

Title of the paper: Engineering Mathematics- III/Applied Mathematics

Semester: III

Max. Marks: 80

Sub.Code: 301/303 (2001/2002/2003/2004/2005)

Time: 3 Hours

Date: 04-05-2007

Session: AN

PART – A

(10 x 2 = 20)

Answer ALL the Questions

1. Find the Laplace transform of $\frac{e^{at} - e^{bt}}{t}$.
2. Find $L^{-1}\left(\frac{1}{S^2 - 6S + 10}\right)$
3. Solve using Laplace transforms $\frac{dy}{dt} - 3y = e^{2t}$
4. Using Laplace transforms, solve $y + \int_0^t y(t)dt = e^{-t}$
5. Find the value of m such that $2x - x^2 + my^2$ may be harmonic.
6. Find the invariant points of the transformation $W = \frac{1}{z - 2i}$
7. Evaluate $\int_C \frac{dz}{(z-3)^2}$ where C is the circle $|z| = 1$.
8. Find the Residue of $f(z) = \frac{1 + e^z}{z \cos z + \sin z}$ at $z = 0$.
9. Explain Null hypothesis and Alternative hypothesis.
10. What are the assumptions for student's 't' test?

PART – B
Answer All the Questions

(5 x 12 = 60)

11. (a) Find the Laplace transform of $t^2 e^{-2t} \cos t$.
(b) Using convolution theorem find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$.
(or)
12. (a) Find the Laplace Transform of $f(t) = \begin{cases} t & \text{for } 0 < t < 1 \\ 2 - t & \text{for } 1 < t < 2 \end{cases}$
such that $f(t+2) = f(t)$.
(b) Verify the initial and final value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$.
13. Using Laplace transform, solve $\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 2y = 5 \sin t$ given that $y(0) = y'(0) = 0$.
(or)
14. Solve, by using Laplace transform $3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$ and $\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$ given that $x = 0$ and $y = 0$ when $t = 0$.
15. (a) If $f(x)$ is a regular function, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$.
(b) Given that $u = \frac{\sin 2x}{\cosh 2y - \cos 2x}$, find the analytic function $u+iv$?
(or)

16. (a) Construct the analytic function given that $2u+v=e^x[\cos y-sin y]$
- (b) Find the bilinear mapping which maps $-1, 0, 1$ of the z -plane onto $-1,-i,1$ of the w -plane. Show that under this mapping the upper half of the z -plane maps on to the interior of the unit circle $|w| = 1$.

17. (a) Using Cauchy's integral formula, evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle $|z+1-i|=2$.

- (b) Using contour integration, prove that $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos \theta} d\theta = \frac{\pi}{12}$

(or)

18. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$

- (b) Expand $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in Laurent's series if

- (i) $|z| > 3$ (ii) $2 < |z| < 3$

19. (a) The following table gives the frequency of occurrence of the digits 0, 1, ..., 9 in the last place in the four figure logarithm of numbers 10 – 99. Examine if there is any peculiarity

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	6	16	15	10	12	12	3	2	9	5

- (b) The following table gives the length of 12 samples of Egyptian cotton taken from a consignment 48, 46, 49, 46, 52, 45, 43, 47, 47, 46, 45, 50. Test if the mean length of the consignment comes taken as 46.

(or)

20. The nicotine contents in two samples of tobacco are given below:

Sample I:	21	24	25	27	26
Sample II:	22	27	28	31	30

Can you say that the two samples came from the same population?