

ROUGH WORK



**KVPY QUESTION PAPER -STREAM SA**

October 30, 2011

## MATHEMATICS

- 1 Suppose  $a, b, c$  are three distinct real numbers. Let

$$P(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} + \frac{(x-a)(x-b)}{(c-a)(c-b)}.$$

When simplified,  $P(x)$  becomes

- A. 1  
B.  $x$   
C.  $\frac{x^2 + (a+b+c)(ab+bc+ca)}{(a-b)(b-c)(c-a)}$   
D. 0

- 2 Let  $a, b, x, y$  be real numbers such that  $a^2 + b^2 = 81$ ,  $x^2 + y^2 = 121$  and  $ax + by = 99$ . Then the set of all possible values of  $ay - bx$  is

- A.  $\left(0, \frac{9}{11}\right]$   
B.  $\left(0, \frac{9}{11}\right)$   
C.  $\{0\}$   
D.  $\left[\frac{9}{11}, \infty\right)$

3 If  $x + \frac{1}{x} = a$ ,  $x^2 + \frac{1}{x^3} = b$ , then  $x^3 + \frac{1}{x^2}$  is

- A.  $a^3 + a^2 - 3a - 2 - b$
- B.  $a^3 - a^2 - 3a + 4 - b$
- C.  $a^3 - a^2 + 3a - 6 - b$
- D.  $a^3 + a^2 + 3a - 16 - b$

4 Let  $a, b, c, d$  be real numbers such that  $|a-b| = 2$ ,  $|b-c| = 3$ ,  $|c-d| = 4$ . Then the sum of all possible values of  $|a-d|$  is

- A. 9
- B. 18
- C. 24
- D. 30

5 Below are four equations in  $x$ . Assume that  $0 < r < 4$ . Which of the following equations has the largest solution for  $x$ ?

- A.  $5\left(1 + \frac{r}{\pi}\right)^x = 9$
- B.  $5\left(1 + \frac{r}{17}\right)^x = 9$
- C.  $5(1+2r)^x = 9$
- D.  $5\left(1 + \frac{1}{r}\right)^x = 9$

6 Let  $ABC$  be a triangle with  $\angle B = 90^\circ$ . Let  $AD$  be the bisector of  $\angle A$  with  $D$  on  $BC$ . Suppose  $AC = 6$  cm and the area of the triangle  $ADC$  is 10 cm<sup>2</sup>. Then the length of  $BD$  in cm is equal to

- A.  $\frac{3}{5}$
- B.  $\frac{3}{10}$
- C.  $\frac{5}{3}$
- D.  $\frac{10}{3}$

7 A piece of paper in the shape of a sector of a circle (see Fig. 1) is rolled up to form a right-circular cone (see Fig. 2). The value of the angle  $\theta$  is

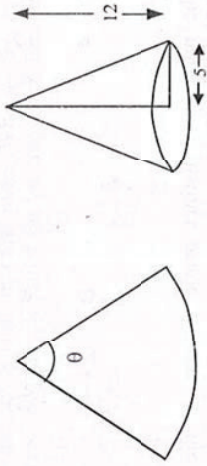
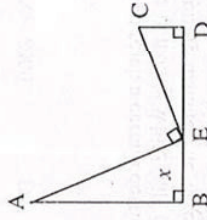


Fig. 1

Fig. 2

- A.  $\frac{10\pi}{13}$
- B.  $\frac{9\pi}{13}$
- C.  $\frac{5\pi}{13}$
- D.  $\frac{6\pi}{13}$

8 In the adjoining figure  $AB = 12$  cm,  $CD = 8$  cm,  $BD = 20$  cm;  $\angle ABD = \angle AEC = \angle EDC = 90^\circ$ . If  $BE = x$ , then



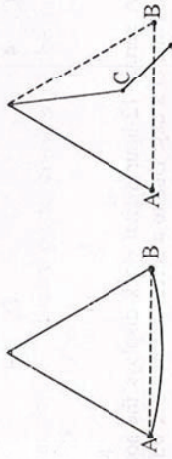
- A.  $x$  has two possible values whose difference is 4
- B.  $x$  has two possible values whose sum is 28
- C.  $x$  has only one value and  $x \geq 12$
- D.  $x$  cannot be determined with the given information



- 9 Three circles each of radius 1 touch one another externally and they lie between two parallel lines. The minimum possible distance between the lines is
- A.  $2 + \sqrt{3}$       B.  $3 + \sqrt{3}$   
 C. 4      D.  $2 + \frac{1}{\sqrt{3}}$
- 10 The number of distinct prime divisors of the number  $512^3 - 25^3 - 259^3$  is
- A. 4      B. 5  
 C. 6      D. 7
- 11 Consider an incomplete pyramid of balls on a square base having 18 layers; and having 13 balls on each side of the top layer. Then the total number  $N$  of balls in that pyramid satisfies
- A.  $9000 < N < 10000$       B.  $8000 < N < 9000$   
 C.  $7000 < N < 8000$       D.  $10000 < N < 12000$
- 12 A man wants to reach a certain destination. One-sixth of the total distance is muddy while half the distance is tar road. For the remaining distance he takes a boat. His speed of traveling in mud, in water, on tar road is in the ratio 3:4:5. The ratio of the durations he requires to cross the patch of mud, stream and tar road is
- A.  $\frac{1}{2} : \frac{4}{3} : \frac{5}{2}$       B. 3:8:15  
 C. 10:15:18      D. 1:2:3
- 13 A frog is presently located at the origin  $(0, 0)$  in the  $xy$ -plane. It always jumps from a point with integer coordinates to a point with integer coordinates moving a distance of 5 units in each jump. What is the minimum number of jumps required for the frog to go from  $(0, 0)$  to  $(0, 1)$ ?
- A. 2      B. 3  
 C. 4      D. 9
- 14 A certain 12-hour digital clock displays the hour and the minute of a day. Due to a defect in the clock whenever the digit 1 is supposed to be displayed it displays 7. What fraction of the day will the clock show the correct time?
- A.  $\frac{1}{2}$       B.  $\frac{5}{8}$   
 C.  $\frac{3}{4}$       D.  $\frac{5}{6}$
- 15 There are 30 questions in a multiple-choice test. A student gets 1 mark for each unattempted question, 0 mark for each wrong answer and 4 marks for each correct answer. A student answered  $x$  questions correctly and scored 60. Then the number of possible value of  $x$  is
- A. 15      B. 10  
 C. 6      D. 5

## PHYSICS

- 16 A simple pendulum oscillates freely between points A and B.

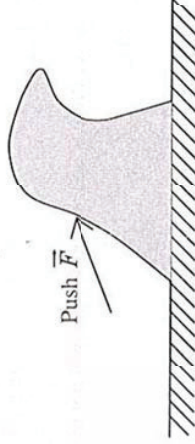


We now put a peg (nail) at some point C as shown. As the pendulum moves from A to the right, the string will bend at C and the pendulum will go to its extreme point D. Ignoring friction, the point D

- A. will lie on the line AB
- B. will lie above the line AB
- C. will lie below the line AB
- D. will coincide with B

8

- 17 A small child tries to move a large rubber toy placed on the ground. The toy does not move but gets deformed under her pushing force ( $\vec{F}$ ) which is obliquely upward as shown. Then



- A. the resultant of the pushing force ( $\vec{F}$ ), weight of the toy, normal force by the ground on the toy and the frictional force is zero
- B. the normal force by the ground is equal and opposite to the weight of the toy
- C. the pushing force ( $\vec{F}$ ) of the child is balanced by the equal and opposite frictional force
- D. the pushing force ( $\vec{F}$ ) of the child is balanced by the total internal force in the toy generated due to deformation

- 18 A juggler tosses a ball up in the air with initial speed  $u$ . At the instant it reaches its maximum height  $H$ , he tosses up a second ball with the same initial speed. The two balls will collide at a height

- A.  $\frac{H}{4}$
- B.  $\frac{H}{2}$
- C.  $\frac{3H}{4}$
- D.  $\sqrt{\frac{3}{4}} H$

9



19 On a horizontal frictionless frozen lake, a girl (36 kg) and a box (9 kg) are connected to each other by means of a rope. Initially they are 20 m apart. The girl exerts a horizontal force on the box, pulling it towards her. How far has the girl traveled when she meets the box?

- A. 10 m
- B. Since there is no friction, the girl will not move
- C. 16 m
- D. 4 m

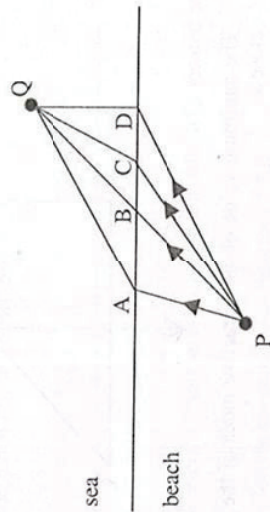
20 The following three objects (1) a metal tray, (2) a block of wood, and (3) a woolen cap are left in a closed room overnight. Next day the temperature of each is recorded as  $T_1$ ,  $T_2$  and  $T_3$  respectively. The likely situation is

- A.  $T_1 = T_2 = T_3$
- B.  $T_3 > T_2 > T_1$
- C.  $T_3 = T_2 > T_1$
- D.  $T_3 > T_2 = T_1$

21 We sit in the room with windows open. Then

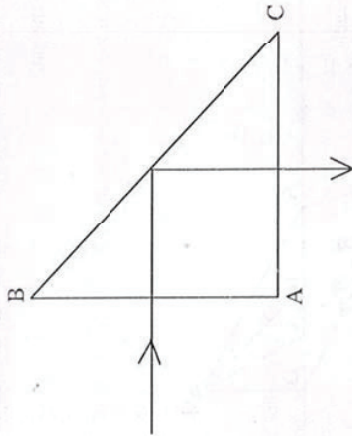
- A. air pressure on the floor of the room equals the atmospheric pressure but the air pressure on the ceiling is negligible
- B. air pressure is nearly the same on the floor, the walls and the ceiling
- C. air pressure on the floor equals the weight of the air column inside the room (from floor to ceiling) per unit area
- D. air pressure on the walls is zero since the weight of air acts downward

22 A girl standing at point P on a beach wishes to reach a point Q in the sea as quickly as possible. She can run at  $6 \text{ km}\cdot\text{h}^{-1}$  on the beach and swim at  $4 \text{ km}\cdot\text{h}^{-1}$  in the sea. She should take the path



- A. PAQ
- B. PBQ
- C. PCQ
- D. PDQ

- 23 Light enters an isosceles right triangular prism at normal incidence through face AB and undergoes total internal reflection at face BC as shown below.



The minimum value of the refractive index of the prism is close to

- A. 1.10      B. 1.55  
C. 1.42      D. 1.72

- 24 A convex lens is used to form an image of an object on a screen. If the upper half of the lens is blackened so that it becomes opaque, then

- A. only half of the image will be visible  
B. the image position shifts towards the lens  
C. the image position shifts away from the lens  
D. the brightness of the image reduces

12

- 25 A cylindrical copper rod has length  $L$  and resistance  $R$ . If it is melted and formed into another rod of length  $2L$ , the resistance will be

- A.  $R$                       B.  $2R$   
C.  $4R$                     D.  $8R$

- 26 Two charges  $+Q$  and  $-2Q$  are located at points A and B on a horizontal line as shown below



The electric field is zero at a point which is located at a finite distance

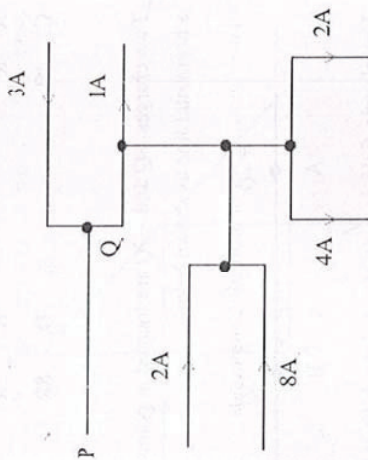
- A. on the perpendicular bisector of AB  
B. left of A on the line  
C. between A and B on the line  
D. right of B on the line

- 27 A 750 W motor drives a pump which lifts 300 litres of water per minute to a height of 6 meters. The efficiency of the motor is nearly (take acceleration due to gravity to be  $10 \text{ m/s}^2$ )

- A. 30%                    B. 40%  
C. 50%                    D. 20%

13

- 28 Figure below shows a portion of an electric circuit with the currents in amperes and their directions. The magnitude and direction of the current in the portion PQ is

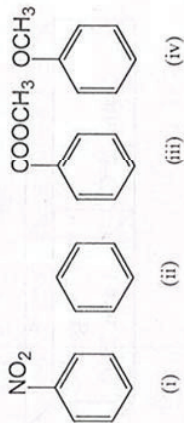


- A. 0 A  
B. 3 A from P to Q  
C. 4 A from Q to P  
D. 6 A from Q to P
- 29 A nucleus of lead  ${}_{82}^{214}\text{Pb}$  emits two electrons followed by an alpha particle. The resulting nucleus will have
- A. 82 protons and 128 neutrons  
B. 80 protons and 130 neutrons  
C. 82 protons and 130 neutrons  
D. 78 protons and 134 neutrons
- 30 The number of air molecules in a  $(5 \text{ m} \times 5 \text{ m} \times 4 \text{ m})$  room at standard temperature and pressure is of the order of
- A.  $6 \times 10^{23}$   
B.  $3 \times 10^{24}$   
C.  $3 \times 10^{27}$   
D.  $6 \times 10^{30}$

## CHEMISTRY

- 31 Two balloons A and B containing 0.2 mole and 0.1 mole of helium at room temperature and 2.0 atm, respectively, are connected. When equilibrium is established, the final pressure of He in the system is
- A. 1.0 atm  
B. 1.5 atm  
C. 0.5 atm  
D. 2.0 atm

- 32 In the following set of aromatic compounds



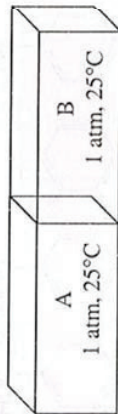
the correct order of reactivity toward Friedel-Crafts alkylation is

- A.  $i > ii > iii > iv$   
B.  $ii > iv > iii > i$   
C.  $iv > ii > iii > i$   
D.  $iii > i > iv > ii$
- 33 The set of principal ( $n$ ), azimuthal ( $l$ ) and magnetic ( $m_l$ ) quantum numbers that is not allowed for the electron in H-atom is
- A.  $n = 3, l = 1, m_l = -1$   
E.  $n = 3, l = 0, m_l = 0$   
C.  $n = 2, l = 1, m_l = 0$   
D.  $n = 2, l = 2, m_l = -1$



- 34 At 298 K, assuming ideal behaviour, the average kinetic energy of a deuterium molecule is
- two times that of a hydrogen molecule
  - four times that of a hydrogen molecule
  - half of that of a hydrogen molecule
  - same as that of a hydrogen molecule

- 35 An isolated box, equally partitioned contains two ideal gases A and B as shown

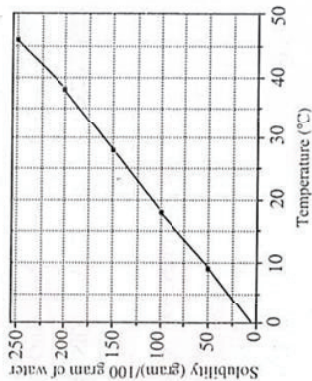


- When the partition is removed, the gases mix. The changes in enthalpy ( $\Delta H$ ) and entropy ( $\Delta S$ ) in the process, respectively, are
- zero, positive
  - zero, negative
  - positive, zero
  - negative, zero

- 36 The gas produced from thermal decomposition of  $(\text{NH}_4)_2\text{Cr}_2\text{O}_7$  is
- oxygen
  - nitric oxide
  - ammonia
  - nitrogen

16

- 37 The solubility curve of  $\text{KNO}_3$  in water is shown below.



- The amount of  $\text{KNO}_3$  that dissolves in 50 g of water at  $40^{\circ}\text{C}$  is closest to
- 100 g
  - 150 g
  - 200 g
  - 50 g

- 38 A compound that shows positive iodoform test is
- 2-pentanone
  - 3-pentanone
  - 3-pentanol
  - 1-pentanol

- 39 After 2 hours the amount of a certain radioactive substance reduces to  $1/16^{\text{th}}$  of the original amount (the decay process follows first-order kinetics). The half-life of the radioactive substance is

- 15 min
- 30 min
- 45 min
- 60 min

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40 In the conversion of a zinc ore to zinc metal, the process of roasting involves

- A.  $ZnCO_3 \rightarrow ZnO$       B.  $ZnO \rightarrow ZnSO_4$   
 C.  $ZnS \rightarrow ZnO$         D.  $ZnS \rightarrow ZnSO_4$

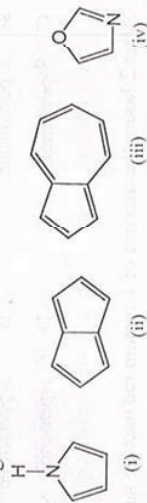
41 The number of P-H bond(s) in  $H_3PO_2$ ,  $H_3PO_3$  and  $H_3PO_4$ , respectively, is

- A. 2, 0, 1                    B. 1, 1, 1  
 C. 2, 0, 0                    D. 2, 1, 0

42 When chlorine gas is passed through an aqueous solution of KBr, the solution turns orange brown due to the formation of

- A. KCl                        B. HCl  
 C. HBr                        D.  $Br_2$

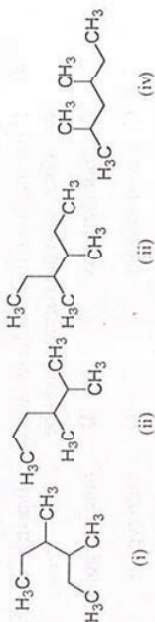
43 Among



the compound which is not aromatic is

- A. i                              B. ii  
 C. iii                            D. iv

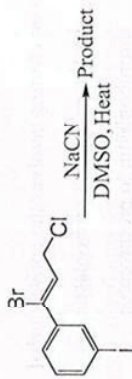
44 Among the following compounds



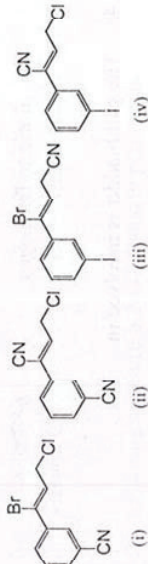
2, 3-dimethylhexane is

- A. i                              B. ii  
 C. iii                            D. iv

45 The major product formed in the reaction



is



- A. i                              B. ii  
 C. iii                            D. iv

## BIOLOGY

- 46 If parents have free ear lobes and the offspring has attached ear lobes, then the parents must be
- A. homozygous
  - B. heterozygous
  - C. co-dominant
  - D. nullizygous
- 47 During meiosis there is
- A. one round of DNA replication and one division
  - B. two rounds of DNA replication and one division
  - C. two rounds of DNA replication and two divisions
  - D. one round of DNA replication and two divisions
- 48 Blood clotting involves the conversion of
- A. prothrombin to thromboplastin
  - B. thromboplastin to prothrombin
  - C. fibrinogen to fibrin
  - D. fibrin to fibrinogen
- 49 The gall bladder is involved in
- A. synthesizing bile
  - B. storing and secreting bile
  - C. degrading bile
  - D. producing insulin
- 50 Which one of the following colours is the LEAST useful for plant life?
- A. red
  - B. blue
  - C. green
  - D. violet
- 51 At rest the volume of air that moves in and out per breath is called
- A. resting volume
  - B. vital capacity
  - C. lung capacity
  - D. tidal volume
- 52 How many sex chromosomes does a normal human inherit from father?
- A. 1
  - B. 2
  - C. 23
  - D. 46
- 53 In the 16th century, sailors who travelled long distances had diseases related to malnutrition, because they were not able to eat fresh vegetables and fruits for months at a time. Scurvy is a result of deficiency of
- A. carbohydrates
  - B. proteins
  - C. vitamin C
  - D. vitamin D
- 54 The following structure is NOT found in plant cells
- A. vacuole
  - B. nucleus
  - C. centriole
  - D. endoplasmic reticulum



55 The cell that transfers information about pain to the brain is called a

- A. neuron
- B. blastocyst
- C. histoblast
- D. haemocyte

56 The presence of nutrients in the food can be tested. Benedict's test is used to detect

- A. sucrose
- B. glucose
- C. fatty acid
- D. vitamins

57 Several minerals such as iron, iodine, calcium and phosphorous are important nutrients. Iodine is found in

- A. thyroxine
- B. adrenaline
- C. insulin
- D. testosterone

58 The principle upon which a lactometer works is

- A. viscosity
- B. density
- C. surface tension
- D. presence of protein

59 Mammalian liver cells will swell when kept in

- A. hypertonic solution
- B. hypotonic solution
- C. isotonic solution
- D. isothermal solution

60 The form of cancer called 'carcinoma' is associated with

- A. lymph cells
- B. mesodermal cells
- C. blood cells
- D. epithelial cells

**MATHEMATICS**

61 Let  $f(x) = ax^2 + bx + c$ , where  $a, b, c$  are integers. Suppose  $f(1) = 0$ ,  $40 < f(6) < 50$ ,  $60 < f(7) < 70$ , and  $1000t < f(50) < 1000(t+1)$  for some integer  $t$ . Then the value of  $t$  is

- A. 2
- B. 3
- C. 4
- D. 5 or more

62 The expression

$$\frac{2^2 + 1}{2^2 - 1} + \frac{3^2 + 1}{3^2 - 1} + \frac{4^2 + 1}{4^2 - 1} + \dots + \frac{(2011)^2 + 1}{(2011)^2 - 1}$$

lies in the interval

- A.  $(2010, 2010\frac{1}{2})$
- B.  $(2011 - \frac{1}{2011}, 2011 - \frac{1}{2012})$
- C.  $(2011, 2011\frac{1}{2})$
- D.  $(2012, 2012\frac{1}{2})$