MSc. Mathematics Degree (MGU-CSS-PG) Examination

MODEL QUESTION PAPER

1st SEMESTER

PC 2 - MT01C02 BASIC TOPOLOGY

PART A

(Answer any five Each question has weightage 1)

Time 3 hrs. Maximum Weight. 30

1. Define: i) Topographical space
   ii) Closure of a set
   iii) Interior point of a set
   iv) Accumulation point of a set

2. If A, B are subsets of a space X show that \( \overline{A \cup B} = \overline{A} \cup \overline{B} \)

3. Let X, Y be topological spaces and \( f : X \to Y \) a function. Show that f is continuous at \( x_0 \in X \) iff for every subset \( A \subset X, x_0 \in \overline{A} \Rightarrow f(x_0) \in \overline{f(A)} \)

4. Define separable space. Prove that every second countable space is separable

5. Let C be a connected subset of a space X and A, B are mutually separated subsets of X. Then \( C \subseteq A \cup B \) implies either \( C \subseteq A \) or \( C \subseteq B \)

6. Prove that components of open subsets of a locally connected space are open

7. Prove that a topological space X is T_1 if and only if every singleton set \( \{ x \} \) is closed in X

8. Prove that compact subsets in Hausdorff space are closed
PART B

(Answer any five Each question has weightage 2)

9. Prove that metrisability is a hereditary property

10. Define derived set of a subset A of space X. Prove that $\overline{A} = A \cup A'$

11. Let $X, Y$ be topological spaces and $f : X \to Y$ a function. If $f$ is continuous then the graph $G = \{(x, f(x)) : x \in X\}$ is homeomorphic to $X$

12. Every continuous real valued function on a compact space is bounded and attains its extrema

13. If $X_1$ and $X_2$ are connected spaces then $X_1 \times X_2$ is connected

14. Every quotient space of a locally connected space is locally connected

15. Prove that all metric spaces are $T_4$

16. If $F$ is a compact subset and $C$ a closed subset of a completely regular space $X$ and $F \cap C = \emptyset$, then there exist a continuous function $f : X \to [0,1]$ such that $f(x) = 0 \forall x \in F$ and $f(y) = 1 \forall y \in C$

PART C

(Answer any three, Each question has weightage 5)

17. a) Every open cover of a second countable space has a countable subcover

b) In a metric space $X$, a point $y$ is in the closure of a subset $A$ iff there exist a sequence $\{x_n\}$ such that $x_n \in A \forall n$ and $\{x_n\}$ converges to $y$ in $X$

18. State and prove Lebesgue covering lemma

19. Let $X$ be a space which is first countable at $x \in X$ and $f : X \to Y$ a function. Then $f$ is
Continuous at x iff for every sequence \( \{ x_n \} \) which converges to x in X, the sequence \( \{ f(x_n) \} \) converges to f(x) in Y

20. A subset of \( \mathbb{R} \) is connected iff it is an interval

21. Every regular, lindelöff space is normal

22. a) In a Hausdorff space limits of sequences are unique

b) Every completely regular space is regular

c) Let Y be a Hausdorff space. Prove that for any space X and any two maps \( f, g : X \to Y \) the set \( \{ x \in X : f(x) = g(x) \} \) is closed in X