INDIAN STATISTICAL INSTITUTE

STUDENTS' BROCHURE

MASTER OF MATHEMATICS
M.MATH.
2003-04

8th MILE, MYSORE ROAD

R.V. COLLEGE P.O.

BANGALORE 560 059

INDIAN STATISTICAL INSTITUTE

STUDENTS' BROCHURE

M.MATH. PROGRAMME

1. GENERAL INFORMATION

1.1. Admission

The details of admission are available in the prospectus. A three–year B. Sc, or a B. E./B. Tech. with Mathematics with an exceptionally strong background in Analysis and Abstract Algebra, or, a B. Stat./B. Math of the Indian Statistical Institute, is eligible for admission to the M.Math. programme. B. Math.(Hons.) Graduates of the Indian Statistical Institute will be eligible for direct admission to the programme. Others have to be admitted through a selection test conducted by the Institute.

Any student who is asked to discontinue the M.Math. programme is not eligible for readmission into this programme even through admission test.

1.2 Duration

The total duration of the M.Math. programme is two years (four semesters). An academic year usually starts in July-August and continues till May, consisting of two semesters with a recess in-between. There is a study-break of one week before the semestral examinations in each semester.

1.3 Courses

Instruction in the M.Math. programme comprises five courses in each of the four semesters. The courses are divided in two groups, one consisting of thirteen compulsory courses and the other, the optional courses where seven have to be chosen out of a list of twenty-two courses. The courses, their sequencing and the syllabi are given later.

1.4 Examinations

The final (semestral) examination in a course is held at the end of the semester. Besides, there is a mid-semestral examination in each course. The calendar for the semester is announced in advance. Usually, the scores of homeworks/assignments, mid-semestral and semestral examinations are combined to get the composite score in a course, as explained in Section 1.5 below. The mid-semestral examinations are held over a period of one week. This examination in each course is normally of a shorter duration than in the semestral examination.

If the composite score (see 1.5) of a student falls short of 45% in a credit course, the student may take a back-paper examination to improve the score. At most one back-paper examination is allowed in each course. Moreover, a student can take at most four back-paper examinations (for credit courses) in the first year and only two in the second year. The decision to allow a student to appear for the back-paper examination is taken by the appropriate Teachers' Committee. The back-paper examination covers the entire syllabus of the course.

If a student misses the mid-semestral or the semestral examination of a course due to medical or family emergency, he/she may be allowed to take a supplementary examination, on an adequately documented representation from the student. The semestral supplementary examination is held at the same time as the back-paper examinations for that semester and a student taking this examination is not given any other examination in the course. (see also 1.5.)

A student may take more than the allotted quota of backpaper examinations in a given academic year, and decide at the end of that academic year which of the BP exam scores should be disregarded.

1.5 Scores

The composite score in a course is a weighted average of the scores in the mid-semestral and semestral examinations, home-assignments, and/or project work in that course; the weights are announced beforehand by the Dean of Studies, or the Class Teacher, in consultation with the teacher concerned .

The minimum composite score to pass a course is 35%.

When a student takes back-paper examination in a credit course, his final score in that course is the higher of the back-paper score and the earlier composite score, subject to a maximum of 45%. When a student takes supplementary mid-semestral or semestral examination in a course, the maximum he/she can score in that examination is 60%. Unlike the back-paper examination, the score in the supplementary examination is used along with other scores to arrive at the composite score.

1.6 Attendance

Each student is required to attend at least 75% of all the classes held in each academic year, failing which he/she is not allowed to appear at the second semestral examination (leading to discontinuation from the programme). (?)

Less than 75% attendance record in the first semester in any academic year leads to reduction of stipend in the following semester; see Section 1.10.

Students with attendance more than 50% but less than 75% in the first semester in any academic year is given warning and urged to improve their attendance. (?)

If a student fails to attend classes in any course continuously for one week or more, he/she would be required to furnish explanation to the Dean of Studies or the Class Teacher for such absence. If such explanation is found to be satisfactory by the Teachers' Committee, then the calculation of percentage of attendance is determined disregarding the period for which explanation has been provided by the student and accepted by the Teachers' Committee.

1.7 Promotion

Here and in what follows, copying in the examination, rowdyism or some other breach of discipline or unlawful/unethical behaviour etc. are regarded as **unsatisfactory conduct.**

A student is considered for promotion to the next year of the programme only when he/she meets the attendance requirement and his/her conduct has been satisfactory. Subject to the above conditions, a student is promoted from First Year to Second Year if the average composite score in all credit courses taken in the first year is not less than 45%, and no composite score in a course is less than 35%. If a student is not promoted from the first year to the second year, he/she has to discontinue the programme.

1.8 Final Result

At the end of the second year the overall average of the percentage composite scores in all the credit courses taken in the two-year programme is computed for each student. The student is awarded the M.Math. degree in one of the following categories according to the criteria he/she satisfies, provided, in the second year, he/she does not have a composite score of less than 35% in a course, and his/her conduct is satisfactory.

Final Result	*Score*
M.Math., First Division with Distinction	The overall average score is at least 75% and the composite score in at most two credit courses is less than 45%.
M.Math., First Division	(i) Not in First Division with Distinction, (ii) the overall average score is at least 60%, and (iii) the composite score in at most four credit courses is less than 45%.
M.Math., Second Division	(i) Not in First Division with Distinction or First Division, (ii) the overall average score is at least 45%, and (iii) the composite score in at most four credit courses is less than 45%.

The students who fail but obtain at least 35% average score in the second year, and have satisfactory conduct are allowed to repeat the final year of the M.Math. programme without stipend; the scores obtained during the repetition of the second year are taken as the final scores in the second year. A student is not given more than one chance to repeat the second year of the programme.

1.9 Class-Teacher

One of the instructors of a class is designated as the Class Teacher. Students are required to meet their respective Class Teachers periodically to get their academic performance reviewed, and to discuss their problems regarding courses.

1.10 Stipends

Stipend, if awarded at the time of admission, is valid initially for the first semester only. The amount of stipend to be awarded in each subsequent semester will depend on academic performance, conduct, and attendance, as specified below, provided the requirements for continuation of the academic programme (excluding repetition) are satisfied; see Sections 1.6 and 1.7.

- *Performance in course work:*
    1. All composite scores used in the following are considered after the respective back-paper examinations.
    2. If all the requirements for continuation of the programme are satisfied, and the average composite score is at least 60% and the number of credit course scores

less than 45% is at most one in any particular semester, the full value of the stipend is awarded in the following semester.

3. If all the requirements for continuation of the programme are satisfied, and the average composite score is at least 45% and the number of credit course scores less than 45% is at most one in any particular semester, then half stipend is awarded in the following semester.

4. In all cases other than (2) and (3) above, no stipend is awarded in the following semester.

- *Attendance*

  1. If the overall attendance in all courses in the first semester in any academic year is less than 75%, no stipend is awarded in the following semester. (?)

- *Conduct*

  The Dean of Studies or the Class Teacher, **at any time**, in consultation with the respective Teachers' Committee, may withdraw the stipend of a student fully for a specific period if his/her conduct in the campus is found to be unsatisfactory.

Note: The net amount of the stipend to be awarded is determined by simultaneous and concurrent application of all clauses described above; but, in no case, the amount of stipend to be awarded or to be withdrawn should exceed 100% of the prescribed amount of stipend.

Stipends can be restored because of improved performance and/or attendance, but no stipend is restored with retrospective effect.

Stipends are given after the end of each month for eleven months in each academic year. The first stipend is given two months after admission with retrospective effect provided the student continues in the M.Math. programme for at least two months.

Contingency grants can be used for purchasing a scientific calculator and other required accessories for the practical class, text books and supplementary text books and for getting photostat copies of required academic material. All such expenditure should be approved by the Class Teacher. No contingency grants are given in the first two months after admission.

1.11 ISI Library Rules

Any student is allowed to use the reading room facilities in the library and allowed access to the stacks. M.Math. students have to pay a security deposit of Rs. 100 in order to avail the borrowing facility. A student can borrow at most four books at a time.

Any book from the Text Book Library (TBL) collection may be issued out to a student only for overnight or week-end provided at least one copy of that book is left in the TBL. Only one TBL book is issued at a time to a student. *Fine is charged if any book is not returned by the due date stamped on the issue-slip*. The library rules, and other details are posted in the library.

1.12 Change of Rules

The Institute reserves the right to make changes in the above rules, course structure and the syllabi as and when needed.

2. COURSE STRUCTURE

2.1 Compulsory Courses (C 1 to C 13)

1. General Topology
2. Complex Analysis
3. Measure theory
4. Algebra I
5. Algebra II
6. Functional Analysis
7. Algebraic Topology
8. Differential Geometry
9. Fourier Analysis
10. Partial Differential Equations I
11. Graph Theory and Combinatorics
12. Advanced Probability
13. Representations of Groups

2.2 Optional Courses (Op 1 to Op 22)

Note: PRQ in parentheses following a course name stands for prerequisites in the course.

1. Algebra III: Commutative Algebra
2. Number Theory
3. Algebraic Geometry (PRQ: Op 1)
4. Algebraic Number Theory (PRQ: Op1)
5. Probability & Stochastic process I: Markov Chains and Markov Processes (PRQ: C12)
6. Probability & Stochastic process II (PRQ: C12)
7. Ergodic Theory
8. Lie Groups & Lie Algebra
9. Partial Differential Equations II (PRQ: C10)
10. Algebraic Groups
11. Algebraic and Differential Topology (PRQ: C7)
12. Advanced Functional Analysis (PRQ: C6)
13. Operator Theory
14. Set Theory
15. Mathematical Logic
16. Theory of Computation
17. Advanced Fluid Dynamic
18. Quantum Mechanics I
19. Quantum Mechanics II (PRQ: Op18)
20. Analytical Mechanics
21. Advanced Linear Algebra
22. Special Topics (to be suggested by the faculty)

2.3 Sequencing of Courses:

First Semester (First Year): Courses C1-C4 and one other course to be chosen by the faculty from the list of courses.

Second Semester (First Year): Courses C5-C8 and one other course to be chosen by the faculty from the list of courses.

Third Semester (Second Year): The Course C13 and four other courses.

No restrictions are envisaged in the courses for the fourth semester, except that all the compulsory courses must be covered within the four semesters and that a course with a prerequisite can be taken by a student only if the prerequisite course has been taken in a previous semester. It is also envisaged that students' option regarding the selection of courses in the second year will be honoured only with the approval of the faculty.

3. DETAILED SYLLABI OF THE COURSES.

C1. General Topology

Part I (9 weeks)

1. Topological spaces, open and closed sets, basis, closure, interior and boundary. Subspace topology, Hausdorff spaces.
2. Continuous maps: properties and constructions; Pasting Lemma. Homeomorphisms. Product topology and Quotient topology (emphasising universal properties).
3. Connected, path-connected and locally connected spaces.
4. Lindelof and Compact spaces, Locally compact spaces and one-point compactification. Tychonoff's theorem.
5. Countability and separation axioms. Urysohn's lemma, Tietze extension theorem and applications.
6. Completion of metric spaces. Baire Category Theorem and applications.
   If time permits:
   (i) Convergence, nets and filters
   (ii) Urysohn embedding lemma and metrization theorem for second countable spaces.
   (iii) Stone-Cech compactification. Paracompactness.

Part II (5 weeks)

1. Constructions of topological manifolds. Projective spaces.
2. Group actions and examples of important orbit spaces. Examples and basic properties of classical groups.
3. Homotopy of paths. The Fundamental Group.
4. Covering spaces, path lifting and homotopy lifting theorems.
5. Fundamental groups of circle, torus, Mobius band etc.
6. Van Kampen theorem and applications.

References:

- J. R. Munkres, *Topology: a first course*. Prentice-Hall, Inc., 1975.

- J. Dugundji, *Topology*. Allyn and Bacon Series in Advanced Mathematics. Allyn and Bacon, Inc., 1978.
- W. S. Massey, *A basic course in algebraic topology*. Graduate Texts in Mathematics, 127. Springer-Verlag, 1991.
- I. M. Singer and J. A. Thorpe, *Lecture notes on elementary topology and geometry*. Undergraduate Texts in Mathematics. Springer-Verlag, 1976.
- K. K. Mukherjea, *Unpublished Notes (Chapter 1).*

C2. Complex Analysis

A review of basic Complex Analysis: Cauchy-Riemann equations, Cauchy's theorem and estimates. power series expansions, maximum modulus principle, Classification of singularities and calculus of residues. Normal families, Arzela's theorem. Product developments, functions with prescribed zeroes and poles, Hadamard's theorem. Conformal mappings, the Riemann mapping theorem, the linear fractional transformations.

Depending on time available, some of the following topics may be done:
(i) Subharmonic functions, the Dirichlet problem and Green's functions
(ii) An introduction to elliptic functions
(iii) Introduction to functions of several complex variables.

References:
- L. V. Ahlfors, *Complex analysis. An introduction to the theory of analytic functions of one complex variable*. McGraw-Hill Book Co., 1978.
- J. B. Conway, *Functions of one complex variable*. II. Graduate Texts in Mathematics, 159. Springer-Verlag, 1995.
- W. Rudin, *Real and complex analysis*. McGraw-Hill Book Co., 1987.

C3. Measure Theory

σ–algebras of sets, measurable sets and measures, extension of measures, construction of Lebesgue measure, integration, convergence theorems, Radon-Nikodym theorem, product measures, Fubini's theorem, differentiation of integrals, absolutely continuous functions (as e.g., in Royden, Chapter 5), L_p-spaces, Riesz representation theorem for the space C[0, 1].

References:
- J. Nevue, *Mathematical foundations of the calculus of probability.* Holden-Day, Inc., 1965.
- I. K. Rana, *An introduction to measure and integration*. Narosa Publishing House, 1997.
- P. Billingsley, *Probability and measure*. John Wiley & Sons, Inc., 1995.
- W. Rudin, *Real and complex analysis*. McGraw-Hill Book Co., 1987.
- K. R. Parthasarathy, *Introduction to probability and measure*. The Macmillan Co. of India, Ltd., 1977.

C4. Algebra I

1. Groups [3-4 weeks]

(A) Review: normal subgroups and quotient groups, homomorphism theorems, direct product, direct sum and free abelian groups (including infinite index) emphasising universal properties. (Categories and functors including universal objects and adjoints, free groups may be introduced).
(B) Group actions on sets and applications (including Sylow theorems and applications).
(C) Permutation groups, simple groups, composition series, solvable and nilpotent groups.
(D) Exact sequences, automorphism and semi-direct product.

2. Rings and Modules [11-12 weeks]
   (A) Review: Universal properties of quotient rings; Noether's isomorphism theorems and applications to non-trivial examples; noetherian rings.
   (B) Basic concepts: submodules, quotients, homomorphisms, isomorphism theorems, generators, annihilators, torsion, direct product and sum, direct summand, free modules, finitely generated modules, noetherian modules. Algebras, finitely generated algebras. Exact and split exact sequences.
   (C) Tensor product of modules and algebras. Tensor, symmetric and exterior algebras.
   (D) Finitely generated modules over a PID: structure theorem and applications to abelian groups.
   (E) Review of topics in Linear Algebra: Matrices and Linear Transformations, Trace, Rank, Determinant, Minimum polynomial, Characteristic Roots and Polynomials. Rational and Jordan Canonical forms. Inner Product Spaces. Unitary, Hermitian and Orthogonal Transformations, Quadratic Forms.

Time permitting, additional topics can be selected from
(i) Snake lemma, complexes, homology sequences.
(ii) Projective and flat modules. Shanuel lemma.

References:

- J. J. Rotman, *An introduction to the theory of groups*. Graduate Texts in Mathematics, 148. Springer-Verlag, 1995.
- N. Jacobson, Basic algebra. Vol. I. W. H. Freeman and Company, 1985.
- S. Lang, *Algebra*. Graduate Texts in Mathematics, 211. Springer-Verlag, 2002.
- N. S. Gopalakrishnan, *University algebra*. Wiley Eastern Ltd., 1986.
- N. S. Gopalakrishnan, *Commutative algebra*. Oxonian Press Pvt. Ltd., 1984.

C5. Algebra II

1. Rings and ideals (Review): operations on ideals (sum, product, quotient and radical); Chinese remainder theorem; nilradical and Jacobson radical. Localisation and local rings. Results on prime ideals like prime avoidance, prime ideals under localisation and theorems of Cohen and Isaac.
2. Modules over local rings. Cayley-Hamilton, NAK lemma and applications. Examples of local-global principles.
3. Polynomial and power series rings: properties and non-trivial applications. Hilbert basis theorem.
4. Algebraic extensions: finite and algebraic field extensions, field automorphisms, existence and uniqueness of algebraic closure, splitting fields and normal extensions, separable, inseparable and purely inseparable extensions, separable and purely inseparable closure, theorem of primitive elements, finite fields.

5. Galois theory: Galois extension and Galois groups, Artin's theorem, fundamental theorem, roots of unity, cyclotomic extensions, linear independence of characters, traces and norms, cyclic extensions, Hilbert theorem 90, Artin-Schreier theorem, algebraic independence of homomorphisms, normal basis theorem.
6. Transcendental extensions: transcendence degree, separating transcendental bases. Derivations, separable extensions, linear disjointness.
7. Integral extensions: integral closure, normalisation and normal rings, Cohen-Seidenberg theorems, Krull dimension, Noether's normalisation, Hilbert's Nullstellensatz and applications, algebraic sets, finiteness of integral closure.

If time permits, topics can be selected from

(i) Galois Cohomology, Kummer Extension.

(ii) Applications and computations: constructions with straight-edge and compasses, solvable and radical extensions; computation of Galois groups, polynomials of degree 3 and 4.

(iii) Real fields: Ordered fields, real closed fields, Sturm theorem, real zeros and homomorphisms.

(iv) Review of PID and UFD. Nagata's criterion for UFD and applications (including Gauss' Theorem); equivalence of PID and one-dimensional UFD.

(v) Weierstrass preparation theorem.

References:

- I. Kaplansky, *Commutative rings*. The University of Chicago Press, 1974.
- S. Lang, *Algebra*. Graduate Texts in Mathematics, 211. Springer-Verlag, 2002.
- M. Nagata, *Field theory*. Pure and Applied Mathematics, No. 40. Marcel Dekker, Inc., 1977.

- H. Matsumura, *Commutative ring theory*. Cambridge Studies in Advanced Mathematics, 8. Cambridge University Press, Cambridge, 1989.
- E. Kunz, *Introduction to commutative algebra and algebraic geometry*. Birkhäuser Boston, Inc., 1985.
- N. S. Gopalakrishnan, *University algebra*. Wiley Eastern Ltd., 1986.
- N. S. Gopalakrishnan, *Commutative algebra*. Oxonian Press Pvt. Ltd., 1984.

C6. Functional Analysis

Normed linear spaces, Banach spaces. Bounded linear operators. Dual of a normed linear space. Hahn-Banach theorem, uniform boundedness principle, open mapping theorem, closed graph theorem. Computing the dual of well-known Banach spaces. Weak and weak* topologies, Banach-Alaoglu Theorem. The double dual, Goldstein's Theorem, reflexivity.
Hilbert spaces, adjoint operators, self-adjoint and normal operators, spectrum, spectral radius, analysis of the spectrum of a compact operator on a Banach space, spectral theorem for bounded self-adjoint, normal, and unitary operators.

References:

- W. Rudin, *Functional analysis*. McGraw-Hill, Inc., 1991.
- J. B. Conway, *A course in functional analysis*. Graduate Texts in Mathematics, 96. Springer-Verlag, 1990.

- K. Yosida, *Functional analysis*. Grundlehren der Mathematischen Wissenschaften, 123. Springer-Verlag, 1980.

C7. Algebraic Topology

Review of C1, Part II, if necessary.

1. Singular homology functors and its axiomatic properties. Relations between fundamental group and first homology. Mayer-Vietoris sequence, computation of homology of spheres. Degree of maps with applications to spheres.
2. Simplicial CW-complexes, cellular description of homology, comparison with singular theory. Computation of homology of projective spaces.
3. Definition of singular cohomology, its fundamental properties, statement of universal coefficient theorem, Betti number and Euler characteristic, cup product, Poincare duality.

Reference:

- M. J. Greenberg, *Lectures on algebraic topology*. W. A. Benjamin, Inc., 1967.
- J. R. Munkres, *Elements of algebraic topology*. Addison-Wesley Publishing Company, 1984.

C8. Differential Geometry

Differentiable manifolds, tangent bundle, vector bundles, vector fields, flows and the fundamental theorem of ODE's. Differential forms and integration, Immersion, submersion, submanifolds and transversality, Riemannian metrics. Riemannian connection on Riemannian manifolds, Gauss-Bonnet Theorem. Parallel transport, geodesics and geodesic completeness, the theorem of Hopf-Rinow.

References:

- F. W. Warner, *Foundations of differentiable manifolds and Lie groups*. Graduate Texts in Mathematics, 94. Springer-Verlag, 1983.
- S. Helgason, *Differential geometry, Lie groups, and symmetric spaces*. Graduate Studies in Mathematics, 34. American Mathematical Society, 2001.

C9. Fourier Analysis

Fourier and Fourier-Stieltjes' series, summability kernels, convergence tests. Fourier transforms, the Schwartz space, Fourier Inversion and Plancherel theorem. Maximal functions and boundedness of Hilbert transform. Paley-Wiener Theorem. Poisson summation formula, Heisenberg uncertainty Principle, Wiener's Tauberian theorem. (An introduction to harmonic analysis on locally compact abelian groups may be given
while discussing Fourier transforms.)
Introduction to wavelets and multi-resolution analysis.

Suggested Texts:

- Y. Katznelson, *An introduction to harmonic analysis*. Dover Publications, Inc., New York, 1976.
- E. Hernández and G. Weiss, *A first course on wavelets*. Studies in Advanced Mathematics. CRC Press, 1996.

C10. Partial Differential Equations-I

Theory of Schwartz distributions and Sobolev spaces; local solvability and Lewy's example; existence of fundamental solutions for constant coefficient differential operators; Laplace, heat and wave equations, hypoelliptic and analytic hypoelliptic operators, elliptic boundary value problems—interior regularity, local existence.
Suggested books:

- G. B. Folland, *Introduction to partial differential equations*. Princeton University Press, 1995.
- F. Trèves, *Basic linear partial differential equations*. Pure and Applied Mathematics, Vol. 62. Academic Press, 1975.
- J. Rauch, *Partial differential equations*. Graduate Texts in Mathematics, 128. Springer-Verlag, 1991.
- E. DiBenedetto, *Partial differential equations*. Birkhäuser Boston, Inc., 1995.
- L. C. Evans, *Partial differential equations*. Graduate Studies in Mathematics, 19. American Mathematical Society, 1998.
- L. Hörmander, *The analysis of linear partial differential operators. I. Distribution theory and Fourier analysis*. Grundlehren der Mathematischen Wissenschaften, 256. Springer-Verlag, 1990.

C11. Graph Theory and Combinatorics

Graphs and digraphs, connectedness, trees, degree sequences, connectivity, Eulerian and hamiltonian graphs, matchings and SDR's, chromatic numbers and chromatic index, planarity, covering numbers, flows in networks, enumeration, Inclusion-exclusion, Ramsey's theorem, recurrence relations and generating functions.
If time permits, some of the following topics may be done: (i) strongly regular graphs, root systems, and classification of graphs with least eigenvalue, (ii) Elements of coding theory (Macwilliams identity; BCH, Golay and Goppa codes, relations with designs).

Suggested texts:

- F. Harary, *Graph Theory*, Addison-Wesley. 1969. Indian Edition available
- D.B. West, *Introduction to Graph Theory*, Prentice - Hall Inc, New Jersey, 1996 (Indian Edition, 1999).
- J.A. Bondy and U.S.R, Murty, *Graph Theory with applications*, Macmillan 1976.
- H.J. Ryser, *Combinatorial Mathematics*, Carus Mathematical Monographs; Math. Assoc. of America, 1963.
- Martin J. Erickson, *Introduction to Combinatorics*, John Wiley & Sons. Inc., NY, 1996.

C12. Advanced Probability

Independence, Kolmogorov Zero-one Law, Kolmogorov Three-series theorem, Strong law of large Numbers. Levy–Cramer Continuity theorem, CLT for i. i. d. components, Infinite Products of probability measures, Kolmogorov's Consistency theorem, Radon–Nikodym Theorem, Conditional expectations.

Discrete parameter martingales with applications.

References:
- J. Nevue, *Mathematical foundations of the calculus of probability.* Holden-Day, Inc., 1965.
- P. Billingsley, *Probability and measure*. John Wiley & Sons, Inc., 1995.
- Y. S. Chow and H. Teicher, *Probability theory. Independence, interchangeability, martingales*. Springer Texts in Statistics. Springer-Verlag, 1997.

C13. Representations of Groups

Structure theory of semisimple rings and modules. Representation of a finite group: Young's Tableaux, examples, Maschke's theorem, sums, products, exterior and symmetric powers of representations. Applications to group rings, characters.

Topological Groups, basic properties like subgroups, quotients and products, fundamental systems of neighbourhoods, open subgroups, connectedness and compactness. Existence of Haar measure on locally compact groups, properties of Haar measures.
Group actions on topological spaces, the space X/G in the topological as also in the analytical case assuming regularity conditions of the group action..

Representation of a locally compact group on a Hilbert space, the associated representation of group algebra, invariant subspaces and irreducibility, Schur's lemma.
Compact groups: Unitarity of finite dimensional representations, Peter-Weyl theory, Representations of SU(2,C)
Induced representation and Frobenius reciprocity theorem, Principal series representations of SL(2, R).

Suggested texts:
- TIFR Lecture Notes on Semisimple rings (Unpublished), Chapters 1 & 4.
- T. Y. Lam, *A first course in noncommutative rings*. Graduate Texts in Mathematics, 131. Springer-Verlag, 2001.
- P. J. Higgins, *Introduction to topological groups*. London Mathematical Society Lecture Note Series, No. 15. Cambridge University Press, 1974.
- L. H. Loomis, *An introduction to abstract harmonic analysis*. D. Van Nostrand Company, Inc., 1953.
- G. B. Folland, *A course in abstract harmonic analysis*. Studies in Advanced Mathematics. CRC Press, 1995.

Op1. Algebra III: Commutative Algebra

1. Free Modules. Projective Modules. Shanuel Lemma. Tensor Product of Modules and Algebras. Tensor, Symmetric and Exterior Algebras. Flat, Faithfully Flat Modules and Finitely Presented Modules.
2. Local-Global Methods. Projective and locally free modules. Patching up of Localisations.
3. Noetherian Modules. Associated Primes and Primary Decomposition. Artininan Modules. Modules of Finite Length.
4. Graded and Filtered Modules. Artin-Rees Theorem.
5. Completion. I-adic Filtrations. Krull Intersection Theorem. Hensel's Lemma and applications. Weierstrass Preparation Theorem.

6. Valuations, Discrete Valuation Rings. Dedekind Domains. Local property of Normal Domains, Normality and DVR at height one primes, Intersection of DVRs. Finiteness of Normalisation. Krull-Akizuki Theorem.
7. Homological Algebra: Complexes, Homology Sequences. Projective Resolution. The functors Tor and Ext.
8. Dimension Theory: Hilbert-Samuel Polynomial. Dimension theorem.
9. Regular Local Rings: Jacobian criterion. UFD criteria: principality of height one primes; Nagata's criterion and applications.
10. (Time permitting) Homological Dimension. Injective Modules and Injective Resolution. Injective Dimension and Global Dimension. Global Dimension of Noetherian Local Rings. Properties of Regular Local Ring. Homological Characterisation of Regular Local Rings. Regular Local Ring is UFD.

Note: The following topics, already included in Algebra I and II, have not been mentioned above. However, if any of these topics have not been covered thoroughly during the previous semesters, they should be covered in this semester.
Operations on Ideals (sum, product, quotient and radical); Chinese remainder theorem; nilradical and Jacobson radical. Localisation and local rings. Results on prime ideals like prime avoidance, prime ideals under localisation and theorems of Cohen and Isaac. Modules over local rings. Cayley-Hamilton and Determinant trick, NAK lemma and applications. Integral extensions: integral closure, normalisation and normal rings, Cohen-Seidenberg theorems.

References:

- H. Matsumura, *Commutative algebra*. W. A. Benjamin, Inc., 1970
- H. Matsumura, *Commutative ring theory*. Cambridge Studies in Advanced Mathematics, 8. Cambridge University Press, Cambridge, 1989.
- D. Eisenbud, *Commutative algebra. With a view toward algebraic geometry*. Graduate Texts in Mathematics, 150. Springer-Verlag, 1995.
- E. Kunz, *Introduction to commutative algebra and algebraic geometry*. Birkhäuser Boston, Inc., 1985.
- J.-P. Serre, *Local algebra*. Springer Monographs in Mathematics. Springer-Verlag, 2000.
- N. S. Gopalakrishnan, *Commutative algebra*. Oxonian Press Pvt. Ltd., 1984.
- *Homological methods in Commutative Algebra*, TIFR Mathematical Pamphlet No. 5, Oxford University Press, 1975.
- M. Reid, *Undergraduate commutative algebra*. London Mathematical Society Student Texts, 29. Cambridge University Press, 1995.
- M. F. Atiyah and I. G. Macdonald, *Introduction to commutative algebra*. Addison-Wesley Publishing Co., 1969.

Op2. Number Theory

Finite fields. Existence and uniqueness of fields of prime power order. Chevalley-Warning theorem on common zeros of systems of polynomial equations over finite fields. Law of quadratic reciprocity.
p-adic fields. p-adic equations and Hensel's Lemma.
Dirichlet series: abscissa of convergence and of absolute convergence. Riemann Zeta function and Dirichlet *L*-functions. Dirichlet's Theorem on primes in arithmetic progression. Functional equation and Euler product for *L*-functions.
Modular forms and the modular group $SL(2, \mathbf{R})$. Eisenstein series. Zeros and poles of modular functions. Dimensions of the spaces of modular forms. The *j*-invariant and Picard's Theorem. *L*-

function and Ramanujan's *T*-function. Jacobi's product formula for *L*-congruence relations satisfied by *T*.

Suggested text: J.-P. Serre, *A course in arithmetic*. Graduate Texts in Mathematics, No. 7. Springer-Verlag, 1973.
If there is time, Hasse-Minkowski theorem from the same book could be included.

Op3. Algebraic Geometry

1. Polynomial rings
2. Hilbert Basis theorem
3. Noether normalisation lemma
4. Hilbert Nullstellensatz
5. Elementary dimension theory
6. Smoothness
7. Curves
8. Divisors on curves
9. Bezout's theorem
10. Abelian differential
11. Riemann–Roch for curves

References:

- C. Musili, *Algebraic geometry for beginners*. Texts and Readings in Mathematics, 20. Hindustan Book Agency, 2001.
- W. Fulton, *Algebraic curves. An introduction to algebraic geometry*. Advanced Book Classics. Addison-Wesley Publishing Company, Advanced Book Program, 1989.
- K. Kendig, *Elementary algebraic geometry*. Graduate Texts in Mathematics, No. 44. Springer-Verlag, 1977.
- R. Shafarevich, *Basic algebraic geometry. 1. Varieties in projective space*. Springer-Verlag, 1994.
- J. Harris, *Algebraic geometry. A first course*. Graduate Texts in Mathematics, 133. Springer-Verlag, 1995.
- M. Reid, *Undergraduate algebraic geometry*. London Mathematical Society Student Texts, 12. Cambridge University Press, 1988.

Op4. Algebraic Number Theory

Dedekind Domains, Fractional Ideals and Class Group, Prime Decomposition in Number Fields, Finiteness of Class Number, Minkowski's Bound, Dirichlet's Unit Theorem.
Valuations, Completions, Product Formula, Decomposition and Inertia Groups, Artin Map.
Distribution of Ideals in a Number Field, Dedekind Zeta Function and Dirichlet L-functions, Frobenius Density Theorem.
Group Cohomology of Cyclic Groups, First and Second Fundamental Inequalities for Cyclic Extensions, Hasse's Norm Theorem, Artin's Reciprocity Law, Kronecker-Weber Theorem, Existence of the Hilbert Class Field.

Prerequisite: Galois Theory, Commutative Algebra.
Suggested Text: G. J. Janusz, *Algebraic number fields*. Pure and Applied Mathematics, Vol. 55. Academic Press, 1973.

Op5. Probability and Stochastic Processes I

Discrete-time Discrete-state Markov Chains, Classification of States, Recurrence, Transience, Stationary Distribution and Stability, Ergodicity, Reversibility.
Topics From :
(a) Rates of convergence to stationarity, Dirichlet Form and Spectral gap methods
(b) Some Coupling methods with applications
(c) Random walk on Finite Groups
(d) Poisson Processes
(e) Continuous time Markov Chains , Birth-and-death processes

Books:

- S. M. Ross, *Stochastic processes*. John Wiley & Sons, Inc., 1996.
- R. N. Bhattacharya and E. C. Waymire, *Stochastic processes with applications*. A Wiley-Interscience Publication. John Wiley & Sons, Inc., 1990.
- E. Giné, G. R. Grimmett and L. Saloff-Coste, *Lectures on probability theory and statistics*. Lecture Notes in Mathematics, 1665. Springer-Verlag, 1997.

Op6. Probability and Stochastic Processes II

Selected topics from the following:

1. Stationary processes.
2. Markov processes and generators
3. Weak Convergence of probability measures on polish spaces including C[0, 1].
4. Brownian motion; construction, simple properties of paths.
5. Poisson processes.
6. Connections between Brownian Motion / Diffusion and PDE's.

References:

- P. Billingsley, *Convergence of probability measures*. John Wiley & Sons, Inc., 1999.
- K. Ito, *Stochastic processes*. Lecture Notes Series, No. 16 Matematisk Institut, Aarhus Universitet, Aarhus 1969.
- D. Revuz and M. Yor, *Continuous martingales and Brownian motion*. Grundlehren der Mathematischen Wissenschaften, 293. Springer-Verlag, 1999.

Op7. Ergodic Theory

1. Measure preserving systems; examples: Hamiltonian dynamics and Liouville's theorem, Bernoulli shifts, Markov shifts, Rotations of the circle, Rotations of the torus, Automorphisms of the Torus, Gauss transformations, Skew-product.
2. Poincare Recurrence lemma: Induced transformation: Kakutani towers: Rokhlin's lemma. Recurrence in Topological Dynamics, Birkhoff's Recurrence theorem
3. Ergodicity, Weak-mixing and strong-mixing and their characterisations
4. Ergodic Theorems of Birkhoff and Von Neumann. Consequences of the Ergodic theorem. Invariant measures on compact systems, Unique ergodicity and equidistribution. Weyl's theorem.
5. The Isomorphism problem; conjugacy, spectral equivalence.
6. Transformations with discrete spectrum, Halmos–von Neumann theorem.

7. Entropy. The Kolmogorov-Sinai theorem. Calculation of Entropy. The Shannon Mc-Millan–Breiman Theorem.
8. Flows. Birkhoff's ergodic Theorem and Wiener's ergodic theorem for flows. Flows built under a function.

References:

- Peter Walters, *An introduction to ergodic theory*. Graduate Texts in Mathematics, 79. Springer-Verlag, 1982.
- Patrick Billingsley, *Ergodic theory and information*. Robert E. Krieger Publishing Co., 1978.
- M. G. Nadkarni, *Basic ergodic theory*. Texts and Readings in Mathematics, 6. Hindustan Book Agency, 1995.
- H. Furstenberg, *Recurrence in ergodic theory and combinatorial number theory*. Princeton University Press, 1981.
- K. Petersen, *Ergodic theory*. Cambridge Studies in Advanced Mathematics, 2. Cambridge University Press, 1989.

Op8. Lie Groups and Lie algebras

1. Linear Lie groups: the exponential map and the Lie algebra of linear Lie group, some calculus on a linear Lie group, invariant differential operators, finite dimensional representations of a linear Lie group and its Lie algebra. Examples of linear Lie group and their Lie algebras e. g Complex groups: GL(n, C), SL(n, C), SO(n, C), Groups of real matrices in those complex groups: GL(n, R), SL(n, R), SO(n, R), Isometry groups of Hermitian forms SO(m, n), U(m, n), SU(m, n). Finite dimensional representations of su(2) and SU(2) and their connection. Exhaustion using the lie algebra su(2).
2. Lie algebras in general, Nilpotent, solvable, semisimple Lie algebra, ideals, Killing form, Lie's and Engel's theorem. Universal enveloping algebra and Poincare-Birkhoff-Witt Theorem (without proof).
3. Semisimple Lie algebra and structure theory: Definition of Linear reductive and linear semisimple groups. Examples of Linear connected semisimple/ reductive Lie groups along with their Lie algebras (look back at 2 above and find out which are reductive/semisimple). Cartan involution and its differential at identity; Cartan decomposition g=k+p, examples of k and p for the groups discussed above. Definition of simple and semisimple Lie algebras and their relation, Cartan's criterion for semisimplicity. Global Cartan decomposition, Root space decomposition; Iwasawa decomposition; Bruhat decomposition (statement only).
4. If time permits, one of the following topics :
   - (i) A brief introduction to Harmonic Analysis on SL (2, R).
   - (ii) Representations of Compact Lie Groups and Weyl Character Formula
   - (iii) Representations of Nilpotent Lie Groups

Suggested Texts:

- J. E. Humphreys, *Introduction to Lie algebras and representation theory*. Graduate Texts in Mathematics, 9. Springer-Verlag, 1978.
- S. C. Bagchi, S. Madan, A. Sitaram and U. B. Tiwari, *A first course on representation theory and linear Lie groups*. University Press, 2000.
- Serge Lang, *SL(2, R)*. Graduate Texts in Mathematics, 105. Springer-Verlag, 1985.
- W. Knapp, *Representation theory of semisimple groups. An overview based on examples*. Princeton Mathematical Series, 36. Princeton University Press, 1986.

Op9. Partial Differential Equations – II

Topics to be selected by the teacher. A possible list of topics is given below:
Cauchy problem: Cauchy-Kowalevska theorem and Holmgren's uniqueness theorem, properties of hyperbolic polynomials, Cauchy problem for a hyperbolic equation.
Differential operators of constant strength, existence when coefficients are continuous, hypoellipticity, non-uniqueness.
Basic theory of pseudo differential operators, L^2 boundedness, Garding's inequality, elliptic regularity theorem.
Semi-group theory, applications to heat, Schrödinger and wave equations.
Spectra of differential operators, random Schrödinger operators.

Suggested books:

- S. Kesavan, *Topics in functional analysis and applications*. John Wiley & Sons, Inc., 1989.
- L. Hörmander, *The analysis of linear partial differential operators. II. Differential operators with constant coefficients*. Grundlehren der Mathematischen Wissenschaften, 257. Springer-Verlag, 1983.
- L. Hörmander, *The analysis of linear partial differential operators. III. Pseudodifferential operators*. Grundlehren der Mathematischen Wissenschaften, 274. Springer-Verlag, 1985.
- M. E. Taylor, *Pseudodifferential operators*. Princeton Mathematical Series, 34. Princeton University Press, 1981.
- M. W. Wong, *An introduction to pseudo-differential operators*. World Scientific Publishing Co., Inc., 1991.
- E. B. Davies, *Spectral theory and differential operators*. Cambridge Studies in Advanced Mathematics, 42. Cambridge University Press, 1995.
- R. Carmona and J. Lacroix, *Spectral theory of random Schrödinger operators*. Probability and its Applications. Birkhäuser Boston, Inc., 1990.

Op10. Algebraic Groups

Review of background commutative algebra (facts on varieties and morphisms as in chapter 0 of Humphreys's book—1st reference below). Definition of linear algebraic groups and homomorphisms over algebraically closed fields, examples. Orbit-closures under actions, linearity of affine groups. Homogeneous spaces and quotients, Chevalley's theorem. Commutative algebraic groups, diagonalizable groups and algebraic tori. Definition of weights and roots, Weyl group. Unipotent groups, Lie-Kolchin theorem, Structure theorem for connected solvable groups. Definition of reductive and semisimple groups, Borel and parabolic subgroups. Basic facts on complete varieties, Borel's fixed point theorem. Conjugacy of maximal tori, Nilpotency of Cartan subgroups. Density theorem and connectedness of centralisers of tori. Normaliser theorem for parabolics. Regular and singular tori, Structure theorem for groups of semisimple rank one. Structure theorem for reductive groups, Bruhat decomposition, semisimple groups. Tits system, standard parabolics, simplicity proof. Root lattice and weight lattice, semisimple and adjoint groups. Representations and their highest weights.

References:

- J. E. Humphreys, *Linear algebraic groups*. Graduate Texts in Mathematics, No. 21. Springer-Verlag, 1975.
- T. A. Springer, *Linear algebraic groups*. Progress in Mathematics, 9. Birkhäuser, 1981.

- R. Steinberg, *Conjugacy classes in algebraic groups*. Lecture Notes in Mathematics, Vol. 366. Springer-Verlag, 1974.

Op11. Algebraic and Differential Topology

Alexander-Lefschetz duality in topological manifolds. De Rham cohomology of manifolds, de Rham theorem, Stokes theorem. Computation of Cohomology rings of projective spaces, Borsuk-Ulam theorem. Higher homotopy groups, fibration, homotopy exact sequence of a pair and of a fibration. Poincare-Hopf theorem.

References.

- R. Bott and L. W. Tu, *Differential forms in algebraic topology*. Graduate Texts in Mathematics, 82. Springer-Verlag, 1982.
- M. J. Greenberg, *Lectures on algebraic topology*. W. A. Benjamin, Inc., 1967
- F. W. Warner, *Foundations of differentiable manifolds and Lie groups*. Graduate Texts in Mathematics, 94. Springer-Verlag, 1983.

Op 12. Advanced Functional Analysis

Brief introduction to topological vector spaces (TVS) and locally convex TVS. Linear Operators. Uniform Boundedness Principle. Geometric Hahn-Banach theorem and applications (Markov-Kakutani fixed point theorem, Haar Measure on locally compact abelian groups, Liapounov's theorem). Extreme points and Krein-Milman theorem.

In addition, one of the following topics:

(a) Geometry of Banach spaces: vector measures, Radon-Nikodym Property and geometric equivalents. Choquet theory. Weak compactness and Eberlein-Smulian Theorem. Schauder Basis.
(b) Banach algebras, spectral radius, maximal ideal space, Gelfand transform
(c) Unbounded operators, Domains, Graphs, Adjoints, spectral theorem

References:

- N. Dunford and J. T. Schwartz, *Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space*. Interscience Publishers John Wiley & Sons 1963
- Walter Rudin, *Functional analysis*. Second edition. International Series in Pure and Applied Mathematics. McGraw-Hill, Inc., 1991.
- K. Yosida, *Functional analysis*. Grundlehren der Mathematischen Wissenschaften, 123. Springer-Verlag, 1980.
- J. Diestel and J. J. Uhl, Jr., *Vector measures*. Mathematical Surveys, No. 15. American Mathematical Society, 1977.

Op 13. Operator theory

I. Compact operators on Hilbert Spaces.
   a) Fredholm Theory
   b) Index
II. C*-algebras—noncommutative states and representations, Gelfand-Neumark representation theorem
III. Von-Neumann Algebras; Projections, Double Commutant theorem, L^infinity functional Calculus

IV. Toeplitz operators

References:

- W. Arveson, *An invitation to C*-algebras*. Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
- N. Dunford and J. T. Schwartz, *Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space*. Interscience Publishers John Wiley & Sons 1963
- R. V. Kadison and J. R. Ringrose, *Fundamentals of the theory of operator algebras. Vol. I. Elementary theory*. Pure and Applied Mathematics, 100. Academic Press, Inc., 1983.
- V. S. Sunder, *An invitation to von Neumann algebras*. Universitext. Springer-Verlag, 1987.

Op 14. Set Theory

Either (a) or (b):

(a) Descriptive Set Theory

A quick review of elementary cardinal and ordinal numbers, transfinite induction, induction on trees, Idempotence of Souslin operation.
Polish spaces, Baire category theorem, Transfer theorems' Standard Borel spaces, Borel isomorphism theorem, sets with Baire property, Kuratowski-Ulam Theorem. The projective hierarchy and closure properties.
Analytic and coanalytic sets and their regularity properties, separation and reduction theorems, Von Neumann and Kuratowski-Ryll Nardzewski's selection theorems, Uniformization of Borel sets with large and small sections. Kondo's uniformization theorem.

References:

- S. M. Srivastava, *A course on Borel sets*. Graduate Texts in Mathematics, 180. Springer-Verlag, 1998.
- S. Kechris, *Classical descriptive set theory*. Graduate Texts in Mathematics, 156. Springer-Verlag, 1995.

(b) Axiomatic Set Theory

A naive review of cardinal and ordinal numbers including regular and singular cardinals, some large cardinals like inaccessible and measurable cardinals. Martin's Axiom and its consequences. Axiomatic development of set theory upto foundation axiom, Class and Class as models, relative consistency, absoluteness, Reflection principle, Mostowski collapse lemma etc. , non-decidability of large cardinal axioms, Godel's second incompleteness theorem, Godel's constructible universe, Forcing lemma and independence of CH.

References:

- K. Kunen, *Set theory. An introduction to independence proofs*. Studies in Logic and the Foundations of Mathematics, 102. North-Holland Publishing Co., 1980.
- T. Jech, *Set theory*. Academic Press, 1978.

Op15. Mathematical Logic

Propositional Logic, Tautologies and Theorems of propositional Logic, Tautology Theorem.

First Order Logic: First order languages and their structures, Proofs in a first order theory, Model of a first order theory, validity theorems, Metatheorems of a first order theory, e. g. , theorems on constants, equivalence theorem, deduction and variant theorems etc. Completeness theorem, Compactness theorem, Extensions by definition of first order theories, Interpretations theorem, Recursive functions, Arithmatization of first order theories, Godel's first Incompleteness theorem, Rudiments of model theory including Lowenheim-Skolem theorem and categoricity.

References: J. R. Shoenfield, *Mathematical logic*. Addison-Wesley Publishing Co., 1967

Op16. Theory of Computation

1. Automata and Languages: Finite automata, regular languages, regular expressions, equivalence of deterministic and non-deterministic finite automata, minimisation of finite automata, closure properties, Kleene's theorem, pumping lemma and its applications, Myhill-Nerode theorem and its uses. Context-free grammar, context-free languages, Chomsky normal form, closure properties, pumping lemma for CFL, pushdown automata. Context–sensitive languages, Chomsky hierarchy, Closure properties, linear bounded automata.
2. Computability: Computable functions, primitive and partial recursive functions, universality and halting problem, recursive and recursively enumerable sets, parameter theorem, diagonalisation and reducibility, Rice'' theorem and its application, Turing machines and its variants, equivalence of different models of computation and Turing-Church thesis.
3. Complexity: Time complexity of deterministic and non-deterministic Turing machines, P and NP, NP-completeness, Cook's theorem: other NP-complete problems.

Reference:

- N. Cutland, *Computability. An introduction to recursive function theory*. Cambridge University Press, 1980.
- M. D. Davis, Ron Sigal and E. J. Weyuker, *Computability, complexity, and languages. Fundamentals of theoretical computer science*. Academic Press, Inc., 1994.
- J. E. Hopcroft and J. D. Ullman, *Introduction to automata theory, languages, and computation*. Addison-Wesley Publishing Co., 1979.
- H. R. Lewis and C. H. Papadimitriou, *Elements of the theory of computation*, Prentice-Hall, 1981(**).
- M. Sipser, *Introduction to the theory of computation*. (**)
- M. R. Garey and D. S. Johnson, *Computers and intractability. A guide to the theory of NP-completeness*. W. H. Freeman and Co., 1979.

Op17. Advanced Fluid Dynamics

Inviscid Incompressible fluid:

Two–dimensional motion. stream function, complex potential and velocity, sources, sinks. Doublets and their images. Circle theorem, Blasius's theorem, Kutta-Jokowaski theorem. Axi-symmetric motion, Stokes stream function. Image of a source and a sink with respect to a sphere.

Vortex motion, vortex lines and filaments, systems of vortices, rectilinear vortices, vortex pair and doublets. A single infinite row of vortices, Karman's vortex sheet.

Linearised gravity waves, progressive waves in deep and shallow water, stationary waves, energy and group velocity, long waves and their energy, capillary waves.

Inviscid compressible fluid:
First and second law of thermodynamics, polytropic gas and its entropy, adiabatic and isentropic flow, propagation of small disturbances. Mach number, Mach cone, irrotional motion, Bernoulli's Equation, pressure, density and temperature in terms of Mach number. Area–velocity relations in one-dimensional flow, concept of subsonic and supersonic flows. Normal shock-wave, Rankine-Hugonoit and Prandtl's relations in case of a plane shock wave.

Viscous incompressible fluid:
Equations of motion of a viscous fluid, Reynold's number, circulation in a viscous liquid, Flow between parallel plates, flow through pipes of circular, elliptic and annular section under constant pressure gradient. Prandtl's concept of boundary layer.

Suggested Texts:

- L. M. Milne-Thomson, *Theoretical hydrodynamics*. The Macmillan Co., 1960.
- L. D. Landau and E. M. Lifshitz, *Fluid mechanics*. Course of Theoretical Physics, Vol. 6 Pergamon Press, 1959
- H. Lamb, *Hydrodynamics*. Cambridge Mathematical Library. Cambridge University Press, 1993.
- W. H. Besant and A. S. Ramsey, *A treatise of Hydro-mechanics*, Part II, ELBS (**).
- P. K. Kundu, *Fluid mechanics*, Academic Press (**).

Op. 18. Quantum Mechanics I

1. (i) Physical basis of Quantum Mechanics.
   (ii) Old Quantum theory
   (iii) Uncertainty, Complimentarity and Duality
   (iv) Measurement problems.
   (v) Heisenberg and Schrodinger representation.
2. (i) Schrodinger wave equation (ii) Perturbation theory.
3. Problem of two or more degrees of freedom without spherical symmetry; Stark effect.
4. Angular momentum, SU(2) algebra
5. Three-dimensional Schrodinger equation. Problems with spherical symmetry . Hydrogen atom, Spherical Harmonic Oscillator.
6. Scattering problem , differential cross section, phase shift, variational principle, SW transformation, Regge poles.
7. WKB approximation.
8. Particles with spin, Pauli matrices, Pauli-Schrodinger equation. Two and three body problems. Hydrogen atom in electric and magnetic field.
9. Quantum Statistics.

References:

- L. I. Schiff, *Quantum Mechanics.* (**)
- J. J. Sakurai, *Modern Quantum Mechanics.*(**)
- L. D. Landau and E. M. Lifshitz, *Quantum mechanics: non-relativistic theory*. Course of Theoretical Physics, Vol. 3. Pergamon Press Ltd., 1958.

- L. M. Falicov, *Group theory and its physical applications*. The University of Chicago Press, 1966.

Op19. Quantum Mechanics II

1. Non stationary problems
2. Relativistic Dirac equation, Spinors.
3. Scattering by a central force.
4. Radiation theory. Quantization of Schrodinger field. Born approximation.
5. Compton effect (Klein Nishina formula)
6. Bremsstrahlung.
7. Symmetry and conservation laws.
8. Quantum Probability and quantum Statistics.
9. Supersymmetric Quantum Mechanics, SWKB. Path integral method. .

References:
- L. I. Schiff, *Quantum Mechanics.* (**)
- P. A. M. Dirac, *The Principles of Quantum Mechanics*. Oxford, at the Clarendon Press, 1947.
- P. A. M. Dirac, *Spinors in Hilbert space*. Plenum Press, 1974.
- M. E. Rose, *Elementary theory of angular momentum*. John Wiley & Sons, Inc.
- R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path integrals.* (**)
- L. D. Landau and E. M. Lifshitz, *Statistical physics*. Course of Theoretical Physics. Vol. 5. Pergamon Press Ltd., 1958.
- S. Flügge, *Practical quantum mechanics*. Classics in Mathematics. Springer-Verlag, Berlin, 1999.
- H. Weyl, *The theory of groups and Quantum Mechanics*. (**)

Op20. Analytical Mechanics

Generalised coordinates, Lagrange's Equation. Examples of Lagrange's equation. Conservation laws. Motion in a central field. Collision of particles. Small Oscillations. Rotating Coordinate systems. Inertial forces. Dynamics of a rigid body. Hamiltonian Mechanics.

Suggested Texts:
- I. Arnold, *Mathematical methods of classical mechanics*. Graduate Texts in Mathematics, 60. Springer-Verlag, 1978.
- R. Abraham and J. E. Marsden, *Foundations of mechanics*. Second edition, Benjamin/Cummings Publishing Co., Inc., Advanced Book Program, 1978.

Op21. Advanced Linear Algebra

The course will cover topics chosen from the following
Majorization and doubly stochastic matrices. Matrix Decomposition Theorems
(Polar, QR, LR, SVD etc.) and their applications. Perturbation Theory.

Nonnegative matrices and their applications. Wavelets and the Fast Fourier
Transform. Basic ideas of matrix computations.

Suggested Text: R. Bhatia, *Matrix analysis*. Graduate Texts in Mathematics, 169. Springer-Verlag, 1997.