INSTRUCTIONS

A. General :

1. This Question Booklet is your Question Paper.
2. This Question Booklet contains **24** pages and has **100** questions.
3. The Question Booklet **Code** is printed on the right-hand top corner of this page.
4. The Question Booklet contains blank spaces for your rough work. No additional sheets will be provided for rough work.
5. **Clip board, log tables, slide rule, calculator, cellular phone, pager and electronic gadgets in any form are NOT allowed.**
6. Write your **Name** and **Roll Number** in the space provided at the bottom.
7. All answers are to be marked only on the machine gradable Objective Response Sheet **(ORS)** provided along with this booklet, as per the instructions therein.
8. The Question Booklet along with the Objective Response Sheet **(ORS)** must be handed over to the Invigilator before leaving the examination hall.
9. Refer to **Special Instruction/Useful Data** on reverse of this sheet.

B. Filling-in the ORS :

10. Write your Roll Number in the boxes provided on the upper left-hand-side of the **ORS** and darken the appropriate oval under each digit of your Roll Number using a **HB pencil**.
11. Ensure that the **code** on the **Question Booklet** and the **code** on the **ORS** are the same. If the codes do not match, report to the Invigilator immediately.
12. On the lower-left-hand-side of the **ORS**, write your Name, Roll Number, Name of the Test Centre and put your signature in the appropriate box with ball-point pen. Do not write these anywhere else.

C. Marking of Answers on the ORS :

13. Each question has **4 choices** for its answer : (A), (B), (C) and (D). Only **ONE** of them is the correct answer.
14. On the right-hand-side of **ORS**, for each question number, darken with a **HB Pencil,** ONLY one oval corresponding to what you consider to be the most appropriate answer, from among the four choices.
15. There will be **negative marking** for wrong answers.

MARKING SCHEME :

(a) For each correct answer, you will be awarded **3 (Three)** marks.
(b) For each wrong answer, you will be awarded **–1 (Negative one)** mark.
(c) Multiple answers to a question will be treated as a wrong answer.
(d) For each un-attempted question, you will be awarded **0 (Zero)** mark.

Name							
Roll Number							

Special Instructions / Useful Data

R denotes the set of real numbers.

C denotes the set of complex numbers.

$R^n = \{(x_1, x_2, \cdots, x_n) : x_1, x_2 \cdots, x_n \in R\}$.

$C^n = \{(z_1, z_2, \cdots, z_n) : z_1, z_2, \cdots, z_n \in C\}$.

P^T denotes the transpose of a matrix P.

$|P|$ denotes the determinant of a matrix P.

$\oplus$ denotes the XOR Boolean operator.

$\overline{A}$ denotes the complement of a Boolean variable A.

$|x|$ denotes the absolute value of x.

f' denotes the derivative of a function f.

f'' denotes the second-order derivative of a function f.

f_x denotes the partial derivative of f with respect to x.

∇f denotes the gradient of f.

$\vec{a}$ denotes a vector a.

$E(X)$ denotes the expected value of a random variable X.

$Var(X)$ denotes the variance of a random variable X.

LPP stands for linear programming problem.

All **bold faced** vectors are column vectors.

For all C programs assume that all standard library functions are accessible.

DO NOT WRITE ON THIS PAGE

Q.1 Consider the following program
```
#include <stdio.h>
main()
{
    int a[2];
    a[0]=a[1]=1;
    switch(a[0])
    {
    case 1: switch(a[1])
            {
                case 1: printf("A");break;
                case 2: printf("B");
            }
    case 2: switch(a[1])
            {
                case 1: printf("A");
                case 2: printf("B");
                default: printf("C");
            }
            break;
    default: printf("D");
    }
}
```

On executing this program, the output is

(A) A   (B) AAB   (C) AD   (D) AABC

Q.2 Consider the following C-program fragment
```
int  s(int n)
{
    int  value = 0;
    while (n > 1) value += n--;
    return value;
}
```

The value returned by the function call s(10) is

(A) 55   (B) 54   (C) 45   (D) 44

Q.3 The value of the arithmetic expression –a*b/c+d%k for integers variables $a=5$, $b=3$, $c=2$, $d=5$, $k=3$ is

(A) –5.5   (B) –5   (C) –3   (D) –1

Q.4 The #include and #define directives are processed at

(A) runtime
(B) compile time and runtime, respectively
(C) compile time
(D) runtime and compile time, respectively

Q.5 Consider the function $f(m) = \begin{cases} 1, & \text{if } m = 1 \text{ or } 2 \\ f(m-1) + f(m-2), & \text{if } m > 2 \end{cases}$. The number of times $f(3)$ is called to compute $f(8)$ is

(A) 10 (B) 8 (C) 7 (D) 6

Q.6 Consider $n+1$ points, $L_0, L_1, \cdots, L_n$, along a straight line such that the distance between L_{k-1} and L_k is k metres. Suppose one ball is placed at each location $L_1, L_2, \cdots, L_n$. A boy standing at L_0 has to pick up each ball and place them in a basket placed at L_0. If the boy cannot carry more than one ball at a time, the minimum distance that the boy travels to place all the balls in the basket is

(A) $n(n+1)$ metres

(B) $\dfrac{n(n+1)(2n+1)}{6}$ metres

(C) $\dfrac{n(n+1)(n+2)}{3}$ metres

(D) $\dfrac{n(n+1)(n+2)}{6}$ metres

Q.7 The number of distinct 3×3 matrices, constructed using nonnegative integers such that each row sum is 3, is

(A) 1000 (B) 729 (C) 300 (D) 99

Q.8 Consider the program fragment

```
float f(float x)
{
    float sum = 0.0;
    float y = x;
    int     n = 1;
    while (y > 0.001)
    {
        sum += y;
        n++;
        y = y*x/n;
    }
    return (sum);
}
```

For a given x, $f(x)$ is approximately equal to

(A) e^x (B) $e^x - 1$ (C) $\ln(1+x)$ (D) $\ln(1-x)$

Q.9 Consider the following statements :

P : If an LPP is infeasible then its dual is unbounded.

Q : An LPP can have more than one optimal solutions.

Which one of the following is TRUE?

(A) Both P and Q are false

(B) Only P is true

(C) Only Q is true

(D) Both P and Q are true

Q.10 A fund manager of a mutual fund wants to buy some number of units of stocks of a software company and of an automobile company. The profit that will be generated over a year per unit of stock of the software company is Rs. 15 and that of the automobile company is Rs. 18. As per the guidelines to the manager, the units of stocks of the software company should be at least twice as much as that of the automobile company. The cost of procurement is Rs. 15 per unit of stock of the software company and Rs. 6 per unit of stock of the automobile company. The manager can use up to Rs. 200 towards the cost of procurement. If the aim is to maximize the profit, then the correct formulation of the LPP is

(A) Maximize $15x_1 + 18x_2$

Subject to $x_1 - 15x_2 \geq 0$

$6x_1 + 2x_2 \geq 200$

$x_1, x_2 \geq 0$

(B) Maximize $15x_1 + 18x_2$

Subject to $2x_1 - x_2 \geq 0$

$15x_1 + 6x_2 \leq 200$

$x_1, x_2 \geq 0$

(C) Maximize $15x_1 + 18x_2$

Subject to $x_1 - 2x_2 \geq 0$

$15x_1 + 6x_2 \leq 200$

$x_1, x_2 \geq 0$

(D) Maximize $15x_1 + 18x_2$

Subject to $x_1 - 2x_2 \geq 0$

$15x_1 + 6x_2 \geq 200$

$x_1, x_2 \geq 0$

Q.11 Let $\mathbf{x}^*$ be an optimal solution to the LPP

Minimize $\mathbf{c}^T\mathbf{x}$

Subject to $A\mathbf{x} \geq \mathbf{b}$

$\mathbf{x} \geq \mathbf{0}.$

Which one of the following is TRUE?

(A) The value of the objective function at a feasible solution to the dual LPP is bounded above by $\mathbf{c}^T\mathbf{x}^*$

(B) $\mathbf{x}^*$ is an extreme point of the feasible region

(C) The dual LPP has an optimal solution with optimum value $\mathbf{c}^T\mathbf{x}^*$

(D) If a variable is zero in $\mathbf{x}^*$ then the corresponding constraint in the dual is satisfied as a strict inequality

Q.12 The time taken by a train in going from A to B and coming back to A at a constant speed 90 km/hour is 7 hours less than the time it takes if it runs at a speed of 100 km/hour in going from A to B and runs at a speed of 50 km/hour in coming back from B to A. The distance between A and B is

(A)   720 km

(B)   820 km

(C)   850 km

(D)   900 km

Q.13 Let $S_1=\{2\}$, $S_2=\{4, 6\}$, $S_3=\{8, 10, 12\}$, $S_4=\{14, 16, 18, 20\}$ and so on. The sum of elements of S_{10} is

(A)   990

(B)   1000

(C)   1010

(D)   1020

Q.14 The value of the C-language expression

$$((x>y)?((x>z)?((y>z)?y:z):x):((y>z)?((x>z)?x:z):y))$$

for $x=5$, $y=8$, $z=6$ is

(A)   0

(B)   5

(C)   6

(D)   8

Q.15 Changing the order of integration of $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{1-x^2} f(x,y)\,dy\,dx$ gives

(A) $\displaystyle\int_{0}^{1}\int_{\sqrt{1-y}}^{\sqrt{1+y}} f(x,y)\,dx\,dy + \int_{-1}^{0}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

(B) $\displaystyle\int_{0}^{1}\int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y)\,dx\,dy - \int_{-1}^{0}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

(C) $\displaystyle\int_{0}^{1}\int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x,y)\,dx\,dy + \int_{-1}^{0}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

(D) $\displaystyle\int_{0}^{1}\int_{\sqrt{1-y}}^{\sqrt{1+y}} f(x,y)\,dx\,dy - \int_{-1}^{0}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

Q.16 Let $W(y_1(x), y_2(x))$ is the Wronskian formed for the solutions $y_1(x)$ and $y_2(x)$ of the differential equation $y''+a_1 y'+a_2 y=0$. If $W \neq 0$ for some $x=x_0$ in $[a, b]$ then

(A) it vanishes for any $x \in [a, b]$

(B) it does not vanish only at $x=a$

(C) it does not vanish for any $x \in [a, b]$

(D) it does not vanish only at $x=b$

Q.17 The general solution of $y''-m^2 y=0$ is

(A) $c_1 \sinh mx + c_2 \cosh mx$

(B) $c_1 \cos mx + c_2 \sin mx$

(C) $c_1 \cos mx + c_2 \sinh mx$

(D) $c_1 \sin mx + c_2 \cosh mx$

Q.18 The general solution of $y y''-(y')^2 =0$ is

(A) $y=c_2 e^{c_1 x^2}$

(B) $y=(c_2+x) e^{c_1 x^2}$

(C) $y=(c_2-x) e^{c_1 x^2}$

(D) $y=c_2 e^{c_1 x}$

Q.19 If $e^x+xy+x\sin y+e^y =c$ is the general solution of an exact differential equation, then the differential equation is

(A) $\dfrac{dy}{dx}=\dfrac{e^x-y-\sin y}{e^y-x-x\cos y}$

(B) $\dfrac{dy}{dx}=\dfrac{e^x+y+\sin y}{e^y+x+x\cos y}$

(C) $\dfrac{dy}{dx}=\dfrac{-(e^x+y+\sin y)}{e^y+x+x\cos y}$
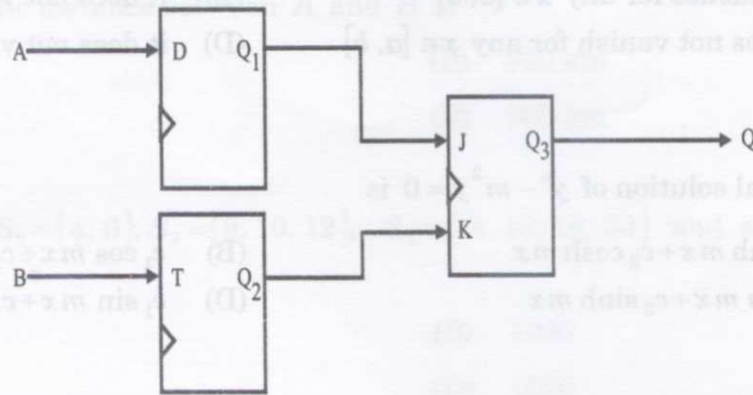
(D) $\dfrac{dy}{dx}=\dfrac{-(e^x-y-\sin y)}{e^y-x-x\cos y}$

Q.20 The general solution of $y'(x+y^2)=y$ is

(A) $x=cy+y^2$

(B) $x=cy-y^2$

(C) $y=cx+x^2$

(D) $y=cx-x^2$

Q.21 The general solution of $y'=2^{x-y}$ is

(A) $2^{-x}+2^{-y}=c$

(B) $2^{-x}-2^{-y}=c$

(C) $2^x+2^y=c$

(D) $2^x-2^y=c$

Q.22 Consider the synchronous circuit given below.

For the current state $Q_1Q_2Q_3 = 000$, which one of the following describes the behaviour of the output Q of the circuit?

(A) Q follows A at the second positive edge of clock

(B) Q follows A at the first positive edge of clock

(C) Q follows B at the first positive edge of clock

(D) Q follows B at the second positive edge of clock

Q.23 Which of the following sets does not contain any C-keyword?

(A) {intern, global, dynamic, local}   (B) {extern, dynamic}

(C) {extern, intern, local, static}   (D) {global, static, dynamic}

Q.24 Internet Explorer and Firefox are

(A) Web portals

(B) WWW browsers

(C) search engines

(D) integrated development environments

Q.25 Which of the following is not an operating system?

(A) Oracle 9.0   (B) Mac OSX   (C) Windows XP   (D) DOS

Q.26 A JK flip-flop is to be implemented using a D flip-flop. Then the input to the D flip-flop is

(A) $JQ + \overline{K}\,\overline{Q}$ (B) $\overline{J}Q + K\overline{Q}$ (C) $\overline{J}\,\overline{Q} + KQ$ (D) $J\overline{Q} + \overline{K}Q$

Q.27 Let P, Q, R be matrices of order 3×5, 5×7 and 7×3, respectively. The number of scalar additions required to compute P(QR) is

(A) 114   (B) 126   (C) 128   (D) 138

Q.28 Consider the set $\{(1, 0, -i), (1+i, 1-i, 1), (i, i, i)\}$ of three vectors from C^3. Which of the following is TRUE about the given set?

(A) Linear span of the set is of dimension 1

(B) Linear span of the set is of dimension 2

(C) Each element of C^3 can be generated as a real linear combination of the elements of the set

(D) The set is a basis of C^3

Q.29 Let $P = \{(x_1, \cdots, x_n) \mid x_1 = 0\}$ and $Q = \{(x_1, \cdots, x_n) \mid x_1 x_2 = 0\}$ be subsets of the vector space R^n. Which one of the following is TRUE about P and Q?

(A) P is not a subspace of R^n and Q is a subspace of R^n

(B) Both P and Q are subspaces of R^n

(C) P is a subspace of R^n and Q is not a subspace of R^n

(D) Neither P nor Q is a subspace of R^n

Q.30 Consider the set $\{1, 3, 7, 9\}$ under the operation of multiplication modulo 10. Which one of the following statements about the given set is FALSE?

(A) It has exactly two elements that are inverses of each other

(B) It is an abelian group

(C) It is a cyclic group

(D) It has a unique generator

Q.31 Which of the following is TRUE for groups of even order?

(A) Such groups do not have a non-trivial proper subgroup

(B) There is no element which is the inverse of itself

(C) The order of such groups is a power of 2

(D) There are at least two elements whose inverses are the elements themselves

Q.32   Which one of the following is NOT a convex set?

(A) $\{x, y \in R : |2x - y| \leq 3, \ |3x + y| \leq 6\}$ (B) $\{x, y \in R : |2x - y| \geq 3, \ 3x + y \geq 6\}$

(C) $\{x, y \in R : 2x - y \leq -3, \ |3x + y| \leq 6\}$ (D) $\{x, y \in R : 2x - y \geq 3, \ 3x + y \leq 6\}$

Q.33   Consider the following LPP

Minimize $2x_1 + 5x_2$

Subject to $3x_1 - 7x_2 \leq 10$

$-2x_1 + x_2 \leq 3$

$x_1 \geq 0, \ x_2$ is unrestricted in sign.

The optimal value of the objective function is

(A) $-\dfrac{50}{7}$ (B) 0 (C) $-\dfrac{207}{11}$ (D)   unbounded

Q.34   Let X and Y be independent Poisson random variables with parameters 5 and μ, respectively. If $P(XY = 1) = P(X + Y = 1)$, then $Var(Y)$ is

(A) $\dfrac{45}{16}$ (B) $\dfrac{5}{2}$ (C) $\dfrac{25}{16}$ (D) $\dfrac{5}{4}$

Q.35   Let X and Y be binomial random variables with parameters (n_1, p_1) and (n_2, p_2), respectively, where $0 < p_1, p_2 < 1, \ p_1 \neq p_2$. Consider the following statements :

P :   If $E(X) = Var(Y)$ then $Var(X) < E(Y)$.

Q :   If $E(X) = E(Y)$ then $Var(X) = Var(Y)$.

Which one of the following is TRUE?

(A)   Both P and Q are false   (B)   Only P is true

(C)   Only Q is true   (D)   Both P and Q are true

Q.36   Let X be a binomial random variable with parameters $(5, p)$. The values of p for which $P(|X - E(X)| \leq 3) = 1$ are given by

(A) $\dfrac{1}{5} \leq p \leq \dfrac{2}{5}$ (B) $\dfrac{2}{5} \leq p \leq \dfrac{3}{5}$ (C) $\dfrac{3}{5} \leq p \leq \dfrac{4}{5}$ (D) $\dfrac{4}{5} \leq p \leq 1$

Q.37 If $u(x,y)=\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{\ln x-\ln y}{x^2+y^2}$, $x,y>0$, then xu_x+yu_y+3u is

(A) u (B) $-u$ (C) 0 (D) -3

Q.38 The volume of the solid of revolution of $y=\dfrac{a}{2}\left(e^{x/a}+e^{-x/a}\right)$ about x-axis between $x=0$ and $x=b$ is

(A) $\dfrac{\pi a^3}{8}\left(e^{2b/a}-e^{-2b/a}\right)-\dfrac{\pi a^2 b}{2}$

(B) $-\dfrac{\pi a^3}{8}\left(e^{2b/a}-e^{-2b/a}\right)+\dfrac{\pi a^2 b}{2}$

(C) $-\dfrac{\pi a^3}{8}\left(e^{2b/a}-e^{-2b/a}\right)-\dfrac{\pi a^2 b}{2}$

(D) $\dfrac{\pi a^3}{8}\left(e^{2b/a}-e^{-2b/a}\right)+\dfrac{\pi a^2 b}{2}$

Q.39 The surface area obtained by revolving $y=2x$, for $x\in[0,2]$, about y-axis is

(A) $2\pi\sqrt{5}$ (B) $4\pi\sqrt{5}$ (C) $2\sqrt{5}\pi$ (D) $4\sqrt{5}\pi$

Q.40 The value of $\displaystyle\iint_D [x+y]\,dx\,dy$, where $[x+y]$ is the greatest integer less than or equal to $x+y$ and D is the region bounded by $x=0$, $y=0$ and $x+y=2$, is

(A) $\dfrac{3}{2}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{4}$ (D) 0

Q.41 What does XP stand for in the operating system Windows XP?

(A) Extra Power   (B) Extended Product

(C) Extra Performance   (D) Experience

Q.42 The next element in the sequence 10, 12, 15, 20, 27, $\cdots$ is

(A) 39   (B) 38   (C) 37   (D) 36

Q.43 If $x=u+v+w$, $y=uv+vw+wu$, $z=uvw$ and $F=F(x,y,z)$ then $uF_u+vF_v+wF_w$ is

(A) $2xF_x+yF_y+3zF_z$

(B) $xF_x+2yF_y+3zF_z$

(C) $2xF_x+3yF_y+zF_z$

(D) $3xF_x+yF_y+2zF_z$

Q.44 The directional derivative of $f(x,y,z)=z^2\,e^{\cos xy}$ at $\left(0,\dfrac{\pi}{2},1\right)$ along $(2,-1,2)$ is

(A) $\dfrac{e}{3}$

(B) $\dfrac{3e}{4}$

(C) $\dfrac{4e}{3}$

(D) $\dfrac{e}{4}$

Q.45 If $f(x)$ and $g(x)$ are continuous in $[a,b]$ and $g(x)\geq 0$ for all $x\in[a,b]$ then $\displaystyle\int_a^b f(x)\,g(x)\,dx$ is

(A) $g(\xi)\displaystyle\int_a^b f(x)\,dx$ for exactly one $\xi\in(a,b)$

(B) $f(\xi)\displaystyle\int_a^b g(x)\,dx$ for all $\xi\in(a,b)$

(C) $f(\xi)\displaystyle\int_a^b g(x)\,dx$ for some $\xi\in(a,b)$

(D) $g(\xi)\displaystyle\int_a^b f(x)\,dx$ for all $\xi\in(a,b)$

Q.46 The value of $\displaystyle\int_0^{\pi/2}\dfrac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ is

(A) $\dfrac{\pi}{2}$

(B) $\dfrac{3\pi}{4}$

(C) $\dfrac{\pi}{4}$

(D) $\dfrac{3\pi}{2}$

Q.47 If $f(x,y)=\cos(x+y)$, then the Taylor's expansion of $f(x,y)$ about $\left(0,\dfrac{\pi}{2}\right)$, up to and including the second degree terms, is

(A) $x+y-\dfrac{\pi}{2}$

(B) $-\left(x+y-\dfrac{\pi}{2}\right)$

(C) $-\left(x+y-\dfrac{\pi}{2}\right)+\dfrac{1}{2}\left(x+y-\dfrac{\pi}{2}\right)^2$

(D) $\left(x+y-\dfrac{\pi}{2}\right)+\dfrac{1}{2}\left(x+y-\dfrac{\pi}{2}\right)^2$

Q.48 The logic circuit shown in the following figure is equivalent to the Boolean expression
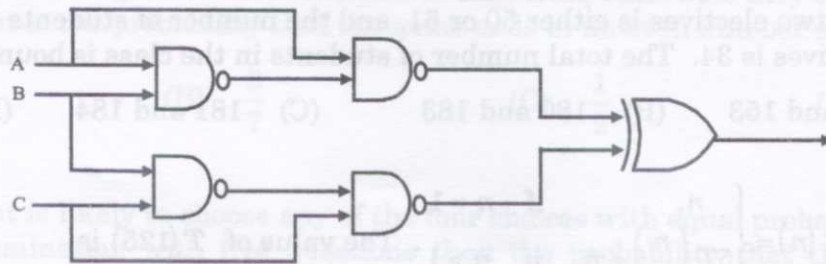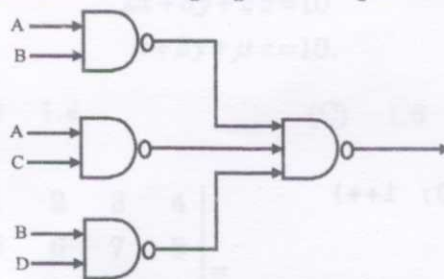
(A) $A \oplus B \oplus C$ (B) $(A \oplus B)\overline{C}$ (C) $(B \oplus C)\overline{A}$ (D) $(A \oplus C)\overline{B}$

Q.49 The logic circuit in the following figure is equivalent to the Boolean expression
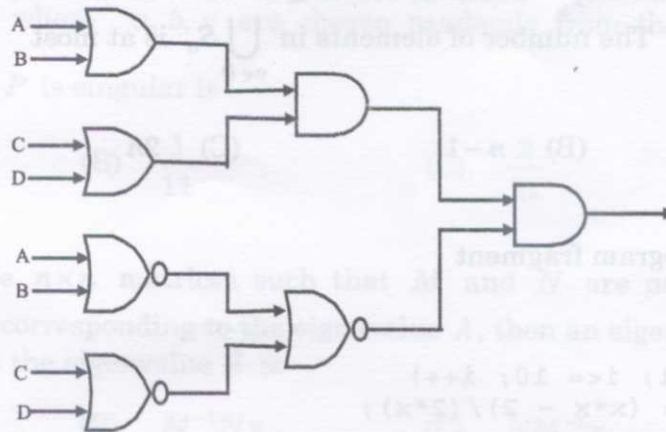
(A) $A+B+C+D$ (B) $AB+CD$ (C) $(A+B)(C+D)$ (D) $ABCD$

Q.50 The logic circuit shown in the following figure is equivalent to the Boolean expression

(A) $\overline{AB} + \overline{AC} + \overline{BD}$

(B) $AB + AC + BD$

(C) $(A+B)(A+C)(B+D)$

(D) $(\overline{A}+\overline{B})(\overline{A}+\overline{C})(\overline{B}+\overline{D})$

Q.51 Each student in a class takes at least one elective out of the three available electives. Each one of the electives is taken by 100 students. The number of students who have taken any two electives is either 50 or 51, and the number of students who have taken all three electives is 34. The total number of students in the class is bounded by

(A) 150 and 153   (B) 180 and 183   (C) 181 and 184   (D) 179 and 182

Q.52 Suppose $T(n) = \begin{cases} n, & \text{if } n = 1 \\ 5T\left(\dfrac{n}{5}\right) + n, & \text{if } n > 1 \end{cases}$. The value of $T(125)$ is

(A) 500   (B) 400   (C) 375   (D) 380

Q.53 Let L be an equivalence relation on a set S of n elements. Consider the set $S_a = \{x : x \neq a \text{ and } aLx\}$. Further, let $D \subseteq S$ be such that no two elements in D are related under L. The number of elements in $\bigcup_{a \in D} S_a$ is at most

(A) $\dfrac{n(n-1)}{2}$ (B) $n-1$ (C) $2n$ (D) n^2

Q.54 Consider the program fragment

```
float x;
x = 1.0;
for(int i=1; i<= 10; i++)
    x -= (x*x - 2)/(2*x);
```

The values of `x` on execution of the above statements is an approximation to

(A) the square root of 5   (B) the square root of 4
(C) the square root of 2   (D) the square root of 10

Q.55 Consider the program

```
#include <stdio.h>
main()
{
    int i,s;
    s = 0;
    for (i=0; i<=10; i++)
        s += i--;
}
```

Which of the following is true about this program?

(A) Execution results in an infinite loop   (B) The program outputs 0
(C) Compilation results in an error   (D) The program outputs 55

Q.56 Three persons play a game by tossing a fair coin each independently. The game ends in a trial if all of them get the same outcome in that trial, otherwise they continue to the next trial. What is the probability that the game ends in an even number of trials?

(A) $\dfrac{2}{7}$ (B) $\dfrac{3}{7}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{4}{7}$

Q.57 If a student is likely to choose any of the four choices with equal probability in a multiple choice examination with five questions then the probability that the student answer at least four questions correctly is

(A) $\dfrac{16}{4^5}$ (B) $\dfrac{15}{4^5}$ (C) $\dfrac{4}{4^5}$ (D) $\dfrac{1}{4^5}$

Q.58 Let $P=\begin{bmatrix}1 & a\\ b & c\end{bmatrix}$, where a, b, c are chosen randomly from the set $\{1, 2, 3, 4, 5\}$. The probability that P is singular is

(A) $\dfrac{2}{25}$ (B) $\dfrac{1}{11}$ (C) $\dfrac{2}{21}$ (D) $\dfrac{1}{5}$

Q.59 Let P, M, N be $n\times n$ matrices such that M and N are nonsingular. If $\mathbf{x}$ is an eigenvector of P corresponding to the eigenvalue λ, then an eigenvector of $N^{-1}MPM^{-1}N$ corresponding to the eigenvalue λ is

(A) $MN^{-1}\mathbf{x}$ (B) $M^{-1}N\mathbf{x}$ (C) $NM^{-1}\mathbf{x}$ (D) $N^{-1}M\mathbf{x}$

Q.60 For which value of μ the following system of equations is inconsistent?

$$3x+2y+z=10$$
$$2x+3y+2z=10$$
$$x+2y+\mu z=10.$$

(A) 0.98   (B) 1.4   (C) 1.6   (D) 1.8

Q.61 If $\begin{vmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{vmatrix}=k$, then $\begin{vmatrix}1 & 2 & 3 & 4\\ 5 & 6 & 7 & 8\\ 9 & 10 & 11 & 12\\ 13 & 14 & 15 & 16\end{vmatrix}=$

(A) $4k+3$ (B) $4k-3$ (C) $2k+1$ (D) k

Q.62 Let $f(x)$ be continuous with $f(0)=1$ and $f\left(\dfrac{\pi}{4}\right)=\dfrac{1}{2}$. If Simpson's $\dfrac{1}{3}$ rule for $\displaystyle\int_0^{\pi/4} f(x)\,dx$

gives k then the value of $f\left(\dfrac{\pi}{8}\right)$ is

(A) $\dfrac{3k}{\pi}-\dfrac{3}{8}$ (B) $\dfrac{8k}{\pi}-\dfrac{3}{4}$ (C) $\dfrac{6k}{\pi}-\dfrac{3}{8}$ (D) $\dfrac{4k}{\pi}-\dfrac{3}{4}$

Q.63 Let $f(x)$ be differentiable. The formula $\dfrac{1}{2h}\left(-3f(x_0)+4f(x_0+h)-f(x_0+2h)\right)$, obtained from quadratic interpolation of $f(x)$, approximates $f'(x)$ at

(A) x_0 (B) x_0+h
(C) x_0+2h (D) any x with $x_0<x<x_0+2h$

Q.64 If the Newton's interpolation formula $f(x)\simeq f(-3)+s(f(3)-f(-3))$ is used to interpolate $f(x)$ at $x=0$, then the value of s is

(A) $\dfrac{3}{2}$ (B) 1 (C) $\dfrac{1}{2}$ (D) 0

Q.65 Let $f(x)$ be continuous, whose values are known at -2, -1, 1 and 2. If the Lagrange's interpolation formula $f(x)=L_1 f(-2)+L_2 f(-1)+L_3 f(1)+L_4 f(2)$ is used to approximate $f(0)$, then L_3 is

(A) 0 (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{4}{3}$

Q.66 The iterative scheme $x_{n+1}=\dfrac{x_n}{2}+\dfrac{3}{x_n}$, with a positive initial approximation, computes

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$

Q.67 Let $f(x)$ be continuous with $f(2)=5$ and $f(7)=-20$. If Regula Falsi (False Position) method is used to find a root of $f(x)=0$, then the first approximation to the root is

(A) 3 (B) 2 (C) 1 (D) 0

Q.68 Which one of the following is TRUE for $f(x, y) = \begin{cases} \dfrac{2x^2 y}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{otherwise} \end{cases}$?

(A) $f_x(0, 0) = 0 = f_y(0, 0)$ and f is differentiable at $(0, 0)$

(B) $f_x(0, 0) = 1 = f_y(0, 0)$ and f is differentiable at $(0, 0)$

(C) $f_x(0, 0) = 0 = f_y(0, 0)$ and f is not differentiable at $(0, 0)$

(D) $f_x(0, 0) = 1 = f_y(0, 0)$ and f is not differentiable at $(0, 0)$

Q.69 If $f_x(0, 0) = 0$, $f_y(0, 0) = 0$, $f_x(0, y) = -y$ and $f_y(x, 0) = x$ then $\left(\dfrac{\partial^2 f}{\partial y \partial x}, \dfrac{\partial^2 f}{\partial x \partial y} \right)$ at $(0, 0)$ is

(A) $(1, 1)$

(B) $\left(1, -\dfrac{1}{2} \right)$

(C) $(-1, 1)$

(D) $\left(1, \dfrac{1}{2} \right)$

Q.70 If $z = e^{xy^2}$, $x = t \cos t$, $y = t \sin t$ then $\dfrac{dz}{dt}$ at $t = \dfrac{\pi}{2}$ is

(A) $\dfrac{\pi^3}{8}$

(B) $\dfrac{\pi^3}{4}$

(C) $\dfrac{\pi^3}{2}$

(D) $-\dfrac{\pi^3}{8}$

Q.71 The function $f(x, y) = \sin x + \sin y + \sin(x + y), 0 < x < \dfrac{\pi}{2}, 0 < y < \dfrac{\pi}{2}$ has

(A) a maximum at $\left(\dfrac{\pi}{4}, \dfrac{\pi}{3} \right)$

(B) a maximum at $\left(\dfrac{\pi}{3}, \dfrac{\pi}{3} \right)$

(C) a minimum at $\left(\dfrac{\pi}{3}, \dfrac{\pi}{3} \right)$

(D) a minimum at $\left(\dfrac{\pi}{4}, \dfrac{\pi}{3} \right)$

Q.72 The Boolean expression $(A \oplus B)(\overline{B} \oplus C)(A \oplus \overline{C})$ is equal to

(A) 0   (B) A (C) B (D) C

Q.73 The Boolean expression $(A+C)(B+\overline{C})$ is equal to

(A) $(A+B+C)(A+\overline{B}+C)(A+B+\overline{C})(\overline{A}+B+\overline{C})$

(B) $(A+B+C)(A+\overline{B}+C)(A+B+\overline{C})(\overline{A}+\overline{B}+\overline{C})$

(C) $(A+B+C)(A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+B+\overline{C})$

(D) $(\overline{A}+B+C)(A+\overline{B}+C)(A+B+\overline{C})(\overline{A}+B+\overline{C})$

Q.74 The Boolean expression $\overline{A} + \overline{A}B + AC$ is equal to

(A) $\overline{A}B+AC$ (B) $A+C$ (C) $\overline{A}+C$ (D) $\overline{A}C$

Q.75 Which one of the following is not equal to the Boolean expression $(\overline{A}+B)(A+\overline{B})$?

(A) $\overline{AB} + A\overline{B}$ (B) $\overline{A}\,\overline{B}+AB$ (C) $\overline{(A+B)(\overline{A}+\overline{B})}$ (D) $\overline{A}B+A\overline{B}$

Q.76 The 2's complement representation of 8-bit signed integers P and Q, using the hexadecimal notation, is 5A and A5 respectively. Which one of the following is FALSE?

(A) $P > Q$ (B) $P+Q=FF$ (C) $P+Q > P$ (D) $P-Q > P$

Q.77 For an unsigned integer, if $(10110001110)_2 = (X)_8 = (Y)_{16}$, then

(A) $X = 2616$ and $Y = 58E$

(B) $X = 6616$ and $Y = D8E$

(C) $X = 5432$ and $Y = B1C$

(D) $X = 5433$ and $Y = B1D$

Q.78 The 8-bit representations for -5 in the sign-magnitude form, 1's complement form and 2's complement form, using hexadecimal notation, are

(A) 85, 8A, 8B   (B) 15, 1A, 1B   (C) 85, FA, FB   (D) F5, FA, FB

Q.79 The range of 1's complement representation of n-bit signed integers is

(A) -2^n to 2^n

(B) $-(2^{n-1}-1)$ to $2^{n-1}-1$

(C) -2^{n-1} to 2^{n-1}

(D) -2^{n-1} to $2^{n-1}-1$

Q.80   Consider the program fragment
```
int *i, a[2];
*i = 5; a[0]=a[1]=1;
i = a;
a[0] = *i;
*(a+1) = *i;
```

The values of `*i`, `a[0]`, and `a[1]` after the execution of the above statements are

(A)   5, 5, 2, respectively

(C)   1, 5, 2, respectively

(B)   1, 1, 1, respectively

(D)   5, 5, 5, respectively

Q.81   Consider the program fragment
```
int f(int x)
{
    static int y = 2;
    if (y==0) return x;
    y--;
    if (x > 0) return f(x-y);
    else return f(x+y);
}
```

The value returned by the function call f(5) is

(A)   1   (B)   2   (C)   3   (D)   4

Q.82   Consider the program fragment
```
int f(int n)
{
    int i = 1;
    n += i;
    {
        int i = 2;
        n += i;
    }
    printf("%d \n", n/i);
    return n;
}
```

The value printed by the function call f(5) is

(A)   8   (B)   6   (C)   4   (D)   3

Q.83 Let $P = \begin{bmatrix} 0 & \omega \\ \omega & 0 \end{bmatrix}$, where ω is a complex cube root of unity. Then P^{24} is

(A) P^2 (B) P (C) Identity matrix (D) 0

Q.84 Consider the system of equations $P\mathbf{x} = 0$, where $P = \begin{bmatrix} 1 & k+4 & 4k+2 \\ 0 & k-2 & -k+2 \\ 0 & k-8 & -5k \end{bmatrix}$. The values of k

for which the system will have a nontrivial solution are

(A) 2 and -2 (B) 2 and -1 (C) -1 and -2 (D) 1 and -1

Q.85 Let P and Q be two $n \times n$ nonzero matrices such that $P + Q = 0$. Which one of the following statements is NEVER true?

(A) P is nonsingular

(B) $P = Q^T$

(C) $P = Q^{-1}$

(D) Rank $(P) \neq$ Rank (Q)

Q.86 Two eigenvalues of the matrix $\begin{bmatrix} 1 & 0 & 0 & -\dfrac{a}{2} \\ 0 & 1 & 0 & -\dfrac{a}{2} \\ 0 & 0 & 1 & -\dfrac{a}{2} \\ 0 & 0 & 0 & a \end{bmatrix}$ are

(A) 1 and a (B) 1 and $-a$ (C) $\dfrac{1}{2}$ and $-\dfrac{a}{2}$ (D) $\dfrac{1}{2}$ and $\dfrac{a}{2}$

Q.87 Consider the function $f(m, n) = \begin{cases} m, & \text{if} \quad n = 1 \\ m + f(m, n-1), & \text{if} \quad n > 1 \end{cases}$. For positive

integers m and n, $f(m, n)$ is

(A) $m + n$ (B) $\displaystyle\sum_{i=0}^{n-1} (m-i)$ (C) m^n (D) mn

Q.88 If the bisection method is used to find a root of $x^3+7x^2-x-7=0$ in the interval $[a,b]$ then a and b are

(A) -6 and -4 (B) -4 and -2 (C) 0 and 2 (D) 4 and 6

Q.89 If $\vec{a},\vec{b}$ and $\lambda\vec{a}-2\vec{b}$ are collinear then the value of λ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.90 If $\left|\vec{a}\right|=1$, $\left|\vec{b}\right|=1$ and the angle between $\vec{a},\vec{b}$ is $\dfrac{\pi}{4}$ then the area of the parallelogram with two adjacent sides as $\vec{a}+5\vec{b}$ and $5\vec{a}-\vec{b}$ is

(A) $13\sqrt{2}$ (B) $12\sqrt{2}$ (C) 13 (D) 12

Q.91 Let $\vec{a}$ and $\vec{b}$ be two non-collinear and non-orthogonal unit vectors. Then $\left|\vec{a}-\left(\vec{a}\bullet\vec{b}\right)\vec{b}\right|$ is

(A) $\left|\vec{a}\times\vec{b}\right|$ (B) $\vec{a}\bullet\vec{b}$ (C) $\left|\vec{a}+\vec{b}\right|$ (D) $\left|\vec{a}-\vec{b}\right|$

Q.92 If $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=3$, $|\vec{b}|=5$ and $|\vec{c}|=8$ then $\vec{a}\bullet\vec{c}$ is

(A) -40 (B) -24 (C) 24 (D) 40

Q.93 If $f(x,y,z)=x-y$ and $\nabla\left(\dfrac{f}{g}\right)=\dfrac{1}{z}\left(\hat{i}-\hat{j}\right)-\left(\dfrac{x-y}{z^2}\right)\hat{k}$ then $g(x,y,z)$ is

(A) xyz (B) x (C) y (D) z

Q.94 The number of bronze medals that India had won in Beijing Olympics 2008 is

(A) 0 (B) 1 (C) 2 (D) 3

Q.95 The area bounded by the curves $y^2 = x$ and $x^2 = y$ is

(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{4}{3}$ (D) $\dfrac{5}{3}$

Q.96 The value of the integral $\displaystyle\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ is

(A) 0   (B) 1   (C) $\dfrac{1}{2}$ (D) -1

Q.97 The speed of light will be minimum while passing through

(A) glass   (B) vacuum
(C) air   (D) water

Q.98 The Vice-President of India is elected by an electoral college consisting of

(A) only the elected members of Lok Sabha and Rajya Sabha
(B) only Rajya Sabha members
(C) all members of Parliament and State Legislative Assemblies
(D) all members of Lok Sabha and Rajya Sabha

Q.99 Match the following :

P.	Main memory	1.	Magnetic memory
Q.	Cache memory	2.	Dynamic random access memory
R.	Hard disk	3.	Optical memory
S.	CDROM	4.	Static random access memory

(A) P-2, Q-4, R-3, S-1   (B) P-1, Q-2, R-3, S-4
(C) P-4, Q-2, R-1, S-3   (D) P-2, Q-4, R-1, S-3

Q.100 Which one of the following pair is NOT a father-child combination?

(A) Lala Amarnath and Mohinder Amarnath
(B) Jagadish Chandra Bose and Satyendra Nath Bose
(C) Ravi Shankar and Anoushka Shankar
(D) Kishore Kumar and Amit Kumar