[This question paper contains 5 printed pages]

Your Roll No

7229 **J**

M.Sc./I

OPERATIONAL RESEARCH

Course I (Basic Mathematics)

(Admissions of 2001 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Five** *questions in all, selecting* **two** *questions from each of sections* **A** *and* **C** *and* **one** *question from section* **B.**

SECTION A

1. (a) Construct the central difference table for the data

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

Hence approximate $f(0.7)$ using Bessel's interpolating polynomial 10

(b) If $f(x) = 1/x^2$, find the divided difference

$$f\,[x_1,\ x_2,\ x_3,\ x_4]$$ 5

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2. (a) Approximate the value of the integral

$$\int_{-3}^{3} x^4 dx$$

Using Weddle's rule, by taking seven equidistant ordinates Compare it with exact value 7

(b) Find an approximation lying in [2, 3] to $\sqrt[3]{25}$ accurate to 10^{-1} using bisection method 8

3 (a) Solve the differential equation

$$\frac{dy}{dx} = (x + y)^{-1}, \quad y(0) = 1$$

for $x =$ 5 (5) 1 by Runge-Kutta method of 4th order 10

(b) Use the Gauss-Jacobi method to solve the following linear system of equations Taking initial approximation $x_1^0 = 0, x_2^0 = 0, x_3^0 = 0$, perform three iterations 5

$$4x_1 + x_2 + 2x_3 = 4$$

$$3x_1 + 5x_2 + x_3 = 7$$

$$x_1 + x_2 + 3x_3 = 3$$

SECTION B

4 Let $B = (b_1, b_2, \quad , b_n)$ be an $n \times n$ non-singular matrix Describe the method to find the inverse of the matrix B in the product form 15

5 (a) Define a convex set Prove that the set of all convex combinations of a finite number of points $x_1, \ldots, x_n$ is a convex set 3

(b) Define a convex cone Prove that the cone generated by a convex set is a convex cone 3

(c) If ω is a boundary point of a closed convex set, then there is at least one supporting hyperplane at ω 9

SECTION C

6 (a) Define a function of bounded variation Show that the function $f \quad [0, 2] \to \mathbb{R}$ defined by

$f(x) = [x]$, the greatest integer not greater than x,

is a function of bounded variation on $[0, 2]$ 5

(b) Prove that a function f is of bounded variation over $[a, b]$ if and only if it can be expressed as the difference of two monotonically increasing functions 5

(c) Show that the sequence $\{f_n\}$ where

$$f_n(x) = x^n$$

is uniformly convergent in [0, K], where K is a number less than 1 and only pointwise convergent in [0, 1] 5

7 (a) Define the Laplace transform of a function f show that

$$L\left(\int_t^{\infty} \frac{e^{-u}}{u}\, du\right) = \frac{\log(s+1)}{s}$$ 6

(b) Prove that if $L^{-1}(F(s)) = f(t)$, then

(i) $L^{-1}(F(a s)) = \frac{1}{a} f\left(\frac{t}{a}\right)$ and

(ii) $L^{-1}\left(e^{-as}F(s)\right) = \begin{cases} f(t-a) & , \ t > a \\ 0 & , \ t < a \end{cases}$

Also evaluate the inverse Laplace transform of

$$\frac{1}{(s-a)(s-b)}$$ 9

8 (a) Solve the following integral equation by stating the conditions under which solution exists

$$u(x) = e^x - \frac{e}{2} + \frac{1}{2} + \frac{1}{2}\int_0^1 u(t)\, dt$$ 7

(b) Define a metric space Prove that any union of open sets in a metric space is open and any finite intersection of open sets in a metric space is open 8