[This question paper contains 4 printed pages]

Your Roll No

7230 **J**

M.Sc./I

OPERATIONAL RESEARCH—Course II

(Statistical Methods)

Time 3 Hours *Maximum Marks 75*

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt any **five** *questions*

1 (a) Discuss the axiomatic approach to probability How does it meet the deficiencies of the classical approach?

(b) A closet contains n pairs of shoes If $2r$ shoes are chosen at random (with $2r < n$), what is the probability that there will be

(i) no complete pair?

(ii) exactly two complete pair among them?

(c) State and prove Bayes theorem 4+6+5

2 (a) If X is a random variable with characteristic function $\phi_X(t)$, and if $\mu'_r = E[X^r]$, exists, then prove that

$$\mu'_r = (-i)^r \left[\frac{\partial^r \phi_X(t)}{\partial t^r} \right]_{t=0}$$

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(b) State and prove Chebychev's inequality.

(c) State and prove Lindeberg-Levy's Central Limit Theorem. 5+5+5

3. (a) Suppose X is a non-negative integer valued random variable. Show that the distribution of X is geometric if it 'lacks memory', i.e., if for each $k \geq 0$ and $Y = X - k$,

$$P[Y = t \mid X \geq k] = P[X = t], t \geq 0$$

(b) If $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ be independent random variables. Find the distribution of X/Y and identify it. 7+8

4. (a) The following results were obtained in the analysis of data on yield of dry bark in ounces (Y) and age in years (X) of 200 cinchona plants:

	X	Y
Average	9.2	16.5
Standard deviation	2.1	4.2

Correlation Coefficient +0.84

Construct the two lines of regression and estimate the yield of dry bark of a plant of age 8 years.

(b) Explain the concept of multiple correlation coefficient $R_{1.23}$, and show that

$$R^2_{1.23} = \frac{r^2_{12} + r^2_{13} - 2r_{12}\,r_{13}\,r_{23}}{1 - r^2_{23}} \leq r^2_{12}$$ 5+10

5 (a) If $X \sim u(0, 1)$, show that $-2\log_e X \sim \chi^2_2$ Hence show that if X_1, X_2 , X_n, are i i d $u(0, 1)$ variates and if $P = X_1, X_2$ X_n, then $-2\log_e P \sim \chi^2_{2n}$

(b) Let X_1 and X_2 be two independent normal variates with the same normal distribution $N(\mu, \sigma^2)$ Obtain the distribution of

$$Y = \frac{X_1 + X_2 - 2\mu}{\sqrt{|X_1 - X_2|^2}}$$

and identify it

(c) Establish the relationship between t and F distribution 6+6+3

6 (a) What do you mean by point estimation? Define the following terms and give one example of each

(i) Consistent estimator,

(ii) Unbiased estimator,

(iii) Sufficient estimator,

(iv) Efficient estimator

(b) For an exponential distribution with mean θ, obtain an unbiased and sufficient estimator for θ based on sample of size n

(c) A random variable X has the p.d.f

$$f(x) = (\beta + 1)x^{\beta}, \; 0 < x < 1, \; \beta > -1$$
$$= 0, \qquad \text{otherwise}$$

obtain the M.L.E. of β. 4+6+5

7 (a) Explain the following terms

(i) Simple and Composite Hypotheses;

(ii) Type I and Type II errors,

(iii) Most Powerful Test,

(iv) Uniformly Most Powerful Test.

(b) If $w = \{x : x \geq 1\}$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from an exponential distribution with mean $1/\theta$, obtain the size of Type I and Type II errors.

(c) In 120 throws of a single die, the following distribution of faces was obtained

Faces	1	2	3	4	5	6	Total
Frequency	30	25	18	10	22	15	120

Fit a suitable distribution and test for goodness of fit 4+5+6