[This question paper contains 5 printed pages]

Your Roll No

7232 **J**

M.Sc./I

OPERATIONAL RESEARCH

Course IV—Linear Programming

(Admissions of 2001 and onwards)

Time 3 *Hours* *Maximum Marks* 75

(Write your Roll No on the top immediately on receipt of this question paper)

Answer any **Five** *questions*

All *questions carry equal marks*

1 (a) Consider a set of linear equations 7

$x_1 + 2x_2 + 4x_3 + x_4 = 7$

$2x_1 - x_2 + 3x_3 - 2x_4 = 4$

Here $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, $x_4 = 0$ is a feasible solution Reduce this feasible solution to a basic feasible solution

(b) Given a basic feasible solution $X_B = B^{-1} b \geq 0$ to the linear programming problem

Max $Z = C^T X$ subject to $AX = b$, $X \geq 0$

Explain how to move from this basic feasible

solution to another basic feasible solution with the improved value of Z 8

2 (a) Explain briefly the standard form I of Revised Simplex Method What are the advantages of the revised Simplex method over the original Simplex method. 5

(b) Use standard form II of Revised Simplex method to solve the following LPP 10

$$\text{Min } Z = 4x_1 + 2x_2 + 3x_3$$

subject to

$$2x_1 + 4x_3 \geq 5$$

$$2x_1 + 3x_2 + x_3 \geq 4$$

$$x_1, x_2, x_3 \geq 0$$

3 (a) State and prove Strong Duality theorem for Duality in the linear programming problem 7

(b) What do you understand by degeneracy in linear programming ? Explain in detail Charne's Perturbation technique to resolve it 8

4 (a) Use Simplex Method for bounded variables to solve the following LPP 9

$$\text{Max } Z = 3x_1 + 5x_2 + 2x_3$$

subject to

$$x_1 + 2x_2 + 2x_3 \leq 14$$

$$2x_1 + 4x_2 + 3x_3 \leq 23$$

$0 \leq x_1 \leq 4$

$2 \leq x_2 \leq 5$

$0 \leq x_3 \leq 3$

(b) Discuss the effect of discrete changes in the Right hand side coefficients of the constraints on the optimal solution of a LPP 6

5 (a) The following table shows all the necessary information on the availability at each warehouse, requirement of each market and the unit cost of transportation from warehouse to the market 9

Market

Warehouse	I	II	III	IV	Availability
A	5	2	4	3	22
B	4	8	1	6	15
C	4	6	7	8	8
Requirement	7	12	17	9	

The shipping clerk has worked out the following schedule 12 units from A to II, 1 unit from A to III, 9 units from A to IV, 15 units from B to III, 7 units from C to I and 1 unit from C to III

(i) Check and see if the clerk has worked out optimal schedule

(ii) Find out the optimal schedule and minimum cost of transportation.

(iii) If in this problem, the transportation cost from C to II is reduced to 5, will it affect the optimal schedule obtained in part (ii)

(b) The building contractor has five jobs A, B, C, D and E and five masons U, V, W, X and Y The number of hours taken by each mason to complete different jobs are as follows 6

	Masons				
	U	V	W	X	Y
A	3	5	9	15	9
B	4	7	14	16	8
Jobs C	7	11	18	19	12
D	4	5	8	15	11
E	9	9	14	22	10

How should the jobs be allocated to the masons to minimize the total time

6 (a) Determine the optimal strategies and the value of the two-person-zero-sum game with payoff matrix as follows 7

$$\begin{bmatrix} 8 & -3 & -5 \\ 2 & 6 & 4 \\ 2 & 3 & 7 \end{bmatrix}$$

(b) Prove the equivalence of a Rectangular game and a Linear programming problem 8

7 (a) Consider the following MOLP

Max $\{Z_1 = -4x_1 + x_2\}$

Max $\{Z_2 = x_1 - x_2\}$

subject to

$$-x_1 + 2x_2 \leq 8$$

$$-x_1 + 2x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

(i) Using the domination set approach, determine which of the points $x^1 = (2, 3)$, $x^2 = (4, 6)$, $x^3 = (0, 3)$ and $x^4 = (7, 7)$ are efficient.

(ii) Solve the weighted-sum lpp using the weight vector $\lambda = (0.6, 0.4)$ 8

(b) Carry out the first iteration of Karmakar's Interior point algorithm for solving the following LPP

$$\text{Min } Z = 2x_1 + x_2$$

subject to

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$ 7