[This question paper contains 4 printed pages]

Your Roll No

7235 **J**

M.Sc./I

OPERATIONAL RESEARCH

Paper VII—Theory of Reliability

(Admissions of 2001 and onwards)

Time 3 *Hours* *Maximum Marks* 75

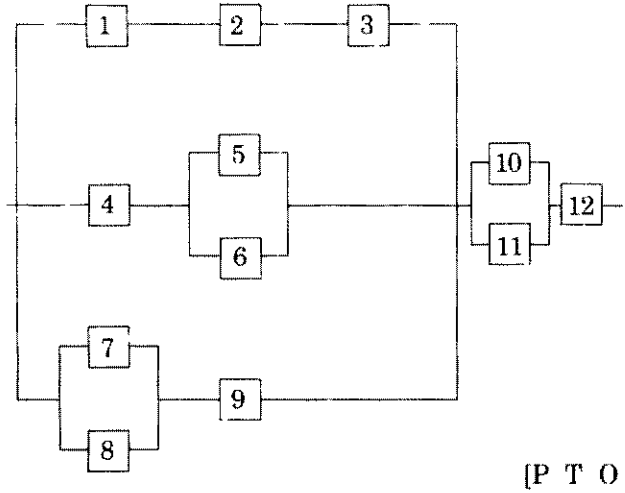
(Write your Roll No on the top immediately on receipt of this question paper)

Attempt **six** *questions in all, selecting* **three** *questions from each section*

Use a separate answer book for each section

SECTION A

1 (a) Consider a system with structure given as follows



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(i) Derive the Structure Function

(ii) Determine the path sets and cut sets.

(iii) Determine the minimal path sets and minimal cut sets

(iv) Express system reliability in terms of minimal path sets and minimal cut sets respectively

(v) Rank the components according to their reliability importance. Justify your answer 10

(b) Describe series and parallel systems. Compare them and explain which arrangement of n components is preferable—series or parallel 2½

2 (a) Derive the reliability function, hazard rate function and MTBF when the system failure time follows Gamma distribution with parameters λ and α, $\lambda, \alpha > 0$ 6½

(b) Obtain the recurrence relationship for determining MTBF for a k-out-of-n G system with exponential failure rate λ for each component. 6

3 (a) Obtain steady-state availability and time dependent state probabilities for a one unit system with constant failure rate 'λ' and constant repair rate 'μ', given that the system is operating at time $t = 0$ with probability p 9½

(b) Explain the difference between System reliability, Maintainability and Availability 3

4 Discuss the steady-state behaviour of a (m, n) system with r repair facilities when each component has the same constant failure rate λ and constant repair rate μ. 12½

SECTION B

5 (a) Define Ordinary, Modified and Equilibrium renewal processes Derive fundamental equation of renewal theory for an ordinary renewal process 6½

(b) Prove that

$$m_r' = \frac{\mu_{r+1}'}{\mu\,(r+1)}$$

where m_r' and μ_r' are the rth moment of limiting distribution of recurrence times and failure time distribution respectively 6

6 (a) Define forward and backward recurrence times Derive expressions for distribution of either of these two times What is probability density function for the equilibrium renewal process 7½

(b) Define a Reward Renewal process and prove that

$$\frac{R(t)}{t} \to \frac{E(R)}{E(X)} \quad \text{as } t \to \infty$$

where the notations have their usual meaning 5

7 What is Minimal Repair ? Discuss the optimal minimal repair policy due to **Barlow and Park** 12½

8 Define Software Rehability Discuss the differences and similarities between Software rehability and Hardware rehability Als discuss in brief the different phases of a Software Development life-cycle 12½