[This question paper contains 4 printed pages]

Your Roll No

7263A **J**

M.Sc. Operational Research/Sem. II

Paper—204 Reliability & Maintenance Theory

Time 3 *Hours* *Maximum Marks* 70

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt any five questions

***All** questions carry equal marks*

1 (a) Define Reliability Function and Hazard Rate Function. If the failure time for a system follows Gamma Distribution, then derive expressions for system Reliability, Mean time before failure and hazard rate function

(b) Define a series-parallel system of order (m, n) and obtain the expression for its reliability and MTBF when all the components have identically and independently distributed exponential failure times

2 Consider a n-unit parallel system with r parallel repair facilities. Let λ and μ denote constant failure and repair rates respectively for each component

Derive system reliability $R(t)$ and system mean time before failure MTBF.

3. Consider a stand by system with n independent and identical components each with an exponential failure time with mean $1/\lambda$. Assume perfect switching, negligible switching time and allow no failure of components while in inactive stand by Allow one repairman who can repair a failed component, one at a time, according to an exponential repair time distribution with mean $1/\mu$ Whenever a component fails, it is immediately subjected to repair if the repairman is idle and otherwise it must wait If all components are down at any time, then as soon as one is repaired, it is immediately placed into operation. Assume all components are initially operable and one component is operating Here $\lambda < \mu$ Now derive

(a) Steady state probability distribution for number of failed components in the system

(b) Expected number of failed components

(c) Steady state availability of the system

(d) Determine the minimum number of components, n_0 in the system such that system availability is atleast P_0

(e) Whenever the repairman is idle, there is a penalty cost of C_1 and whenever the system is in failed condition, there is an additional penalty cost of C_2 Compute the expected total penalty cost at any point of time

4 Consider a system which undergoes preventive maintenance after T hours of continuing operation without failure If the system fails before T hours have elapsed, then emergency repair is performed and preventive maintenance is rescheduled Let the system failure time distribution function be $F(t)$

Derive the condition under which it would be practical to perform preventive maintenance to improve the system availability

Also discuss the case when failure time is exponentially distributed.

5 (a) Define Ordinary, Modified and Equilibrium Renewal processes Prove that for an equilibrium renewal process, the expected number of renewals in any interval is proportional to the length of the time interval

(b) Show that Asymptotic form of renewal function for ordinary renewal process is given by

$$H_0(t)\,,\ \frac{t}{\mu}+\frac{\sigma^2-\mu^2}{2\mu^2}$$

where μ and σ^2 are the mean and variance of failure time

6 (a) What is minimal repair? Discuss the optimal minimal repair policy due to Barlow and Prochan

(b) Consider a renewal process $\{N(t), t \geq 0\}$ with inter renewals time X_n, $n \geq 1$ Whenever a failure occurs, a renewal is said to have taken place

Let $E(X_n) = \mu \; \forall \; n \geq 1$

Then show that with probability one,

$$\frac{N(t)}{t} \to \frac{1}{\mu} \text{ as } t \to \infty$$

7 Consider a one-unit repairable system with failure time and repair time distributions as $F(t)$ and $G(t)$ respectively Using renewal theoretic arguments, show that its uptime Ratio is given by

$$\text{UTR} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$