Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-I & II Remedial Examination Nov/ Dec. 2010

Subject code: 110008 **Subject Name: Maths- I**

Date: 06/12/2010 **Time: 10.30 am – 01.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Attempt the following

(i) Give the geometrical meaning of LMVT. Using LMVT prove that $0 < \frac{1}{x}cos^{-1}\frac{sinx}{x} < 1 , x \in \left[0, \frac{\pi}{2}\right]$ **04**

(ii) If $5x \le f(x) \le 2x^2 + 2$, for all $x \in R$ then find $\lim_{x\to 2} f(x)$ **03**

(b) Attempt the following

(i) Define critical point. If the surface area of a right circular cylinder is given then prove that its height is equal to the diameter of its base when the volume is maximum **04**

(ii) Expand $\sin\left(\frac{\pi}{4} + x\right)$ in powers of x . Hence find the value of $\sin 46^o$ **03**

Q.2 **(a)** Attempt the following

(i) Write the points of nonexistence of a derivative. Prove that **04**

$$f(x) = \begin{cases} x \ , & 0 \le x < \frac{1}{2} \\ 1 \ , & x = \frac{1}{2} \\ 1 - x \ , & \frac{1}{2} < x \le 1 \end{cases}$$

is discontinuous at $x = \frac{1}{2}$

(ii) Check that the sequence $a_n = \frac{n}{n^2+1}$ is a decreasing and bounded below. Is it convergent? **03**

(b) (i) Write the different forms of an improper integrals. Check the convergence of an improper integral $\int_5^{\infty} \frac{7x+4}{x^2+9} dx$ using comparison test **04**

(ii) Evaluate $\int_0^1 x^2 dx$ by finding the Riemann sum, by dividing the interval into unequal subparts. **03**

OR

(b) (i) Evaluate $\int_2^3 (x-2)dx$ by using appropriate area formula. If the range is from 0 to 3 then what will happen? **03**

(ii) Define improper integral. Find the area of between the curve $y^2 = \frac{x^2}{1-x^2}$ and its asymptotes using improper integral. **04**

Q.3 **(a)** If $u = (x^2 + y^2 + z^2)^{\frac{m}{2}}$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ **05**

(b) (i) Use Lagrange method of undetermined multipliers to find the shortest distance from the point (1,2,2) to the sphere $x^2 + y^2 + z^2 = 16$ **04**

(ii)Find the area of the loop of the curve $y^2 = (x-a)(b-x)^2, (b > a)$ **05**

OR

Q.3 **(a)** State modified Euler's theorem. If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 2cos3usinu$ **05**

(b) (i)Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **04**

(ii)Find the volume of the solid of revolution of the area bounded by the curve $y = xe^x$ and the straight lines $x = 1, y = 0$. **05**

Q.4 **(a)** Write the statement of Cauchy's root test. For which values of x does the series $\sum_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)^n x^n, x > 0$ is convergent. What can we say at the point $x = 1$. **04**

(b) (i) Find the values of p for which the series $\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \cdots \infty$ is convergent. **03**

(ii) Test the convergence of $\sum_{n=1}^{\infty}\frac{3^n n!}{n^n}$ **03**

(c) Evaluate $\iint_R e^{2x+3y}dA$ where R is the triangle bounded by $x = 0, y = 0$ and $x + y = 1$. **04**

OR

Q.4 **(a)** Write the statement of Cauchy's integral test. Test the convergence of the series **04**

$$\sum_{n=2}^{\infty}\frac{1}{n(logn)^a}, \quad for\ 0 \leq a \leq 1.$$

(b) (i) Find the interval of convergence for which the series $x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \cdots \infty$ is convergent. **03**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\log(n+1)}$ **03**

(c) Evaluate $\iint_R sin\theta dA$, where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioids $r = 2(1 + cos\theta)$. **04**

Q.5 **(a)** Evaluate $\int_0^1\int_{x^2}^{2-x} xydA$ by changing the order of integration. **04**

(b) (i)Find the directional derivative of the divergence of $\bar{F}(x,y,z) = xyi + xy^2j + z^2k$ at the point (2,1,2) in the direction of the outer normal to the sphere $x^2 + y^2 + z^2 = 9$ **04**

(ii)Use Green's theorem, to evaluate $\oint_C e^{-x}(cosydx - sinydy)$ where C is the rectangle with vertices $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$ **04**

(c) Use L'Hospital rule to find **02**

$$\lim_{x\to\pi/2}\frac{logsinx}{(\pi - 2x)^2}$$

OR

Q.5 **(a)** Evaluate $\iiint_D \sqrt{x^2 + y^2}dV$, where D is the solid bounded by the surfaces $x^2 + y^2 = z^2, z = 0, z = 1$. **04**

(b) (i) A fluid motion is given by $\bar{V} = (ysinz - sinx)i + (xsinz + 2yz)j + (xycosz + y^2)k$. Is the motion irrotational?. If so, find the velocity potential. **04**

(ii)Use divergence theorem to evaluate $\iint_S (x^3dydz + x^2ydzdx + x^2z\,dxdz)$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs $z = 0$ and $z = b$ **04**

Use L'Hospital rule to evaluate **02**

$$\lim_{x\to 1}(1 - x)tan\left(\frac{\pi x}{2}\right)$$

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