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Part III — BUSINESS MATHEMATICS

(English Version)

Time Allowed : 3 Hours]   [Maximum Marks : 200

SECTION – A

N. B. :   i)   Answer *all* the 40 questions.

　　　ii)   Choose and write the correct answer from the four choices given.   40 × 1 = 40

1.   The Adjoint of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is

a)　$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

b)　$\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

c)　$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

d)　$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

2.   If $|A| = 0$ then $|\text{Adj } A|$ is

a)   0

b)   1

c)   – 1

d)   ±1.

[Turn over

3. The rank of an $n \times n$ matrix each of whose elements is 1, is

    a)   1   b)   2

    c) n d) n^2.

4. For what value of k the matrix A, where $A = \begin{pmatrix} 2 & k \\ 3 & 5 \end{pmatrix}$ has no inverse ?

    a) $\dfrac{3}{10}$ b) $\dfrac{10}{3}$

    c)   3   d)   10.

5. A system of linear homogeneous equations has at least

    a)   one solution   b)   two solutions

    c)   three solutions   d)   four solutions.

6. Latus rectum of $y^2 = 4ax$ is

    a) $2a$ b) $3a$

    c) $4a$ d) a.

7. Focus of $y^2 = 16x$ is

    a)   (2, 0)   b)   (4, 0)

    c)   (8, 0)   d)   (2, 4).

8. The semi-major and semi-minor axes of $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$ is

   a)   (4, 5)

   b)   (8, 10)

   c)   (5, 4)

   d)   (10, 8).

9. Eccentricity of the rectangular hyperbola is

   a)   2

   b) $\dfrac{1}{2}$

   c) $\sqrt{2}$

   d) $\dfrac{1}{\sqrt{2}}$.

10. The average fixed cost of the function $c = 2x^3 - 3x^2 + 4x + 8$ is

   a) $\dfrac{2}{x}$

   b) $\dfrac{4}{x}$

   c) $\dfrac{-3}{x}$

   d) $\dfrac{8}{x}$.

11. For the function $y = 3x + 2$ the average rate of change of y when x increases from 1·5 to 1·6 is

   a)   1

   b)   0·5

   c)   0·6

   d)   3.

12. The slope of the normal to the curve $\sqrt{x} + \sqrt{y} = 5$ at (9, 4) is

   a) $\dfrac{2}{3}$

   b) $\dfrac{-2}{3}$

   c) $\dfrac{3}{2}$

   d) $\dfrac{-3}{2}$.

[Turn over

13. The point at which the tangent to the curve $y^2 = x$ makes an angle $\frac{\pi}{4}$ with the x-axis is

   a) $\left(\frac{1}{2}, \frac{1}{4}\right)$

   b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

   c) $\left(\frac{1}{4}, \frac{1}{2}\right)$

   d) $(1, -1)$.

14. The slope of the tangent at $(2, 8)$ on the curve $y = x^3$ is

   a) 3

   b) 12

   c) 6

   d) 8.

15. The stationary value of x for $f(x) = 3(x-1)(x-2)$ is

   a) 3

   b) $\frac{3}{2}$

   c) $\frac{2}{3}$

   d) $\frac{-3}{2}$.

16. If $f(x, y) = 2x + ye^{-x}$ then $f_y(1, 0)$ is

   a) e

   b) $\frac{1}{e}$

   c) e^2

   d) $\frac{1}{e^2}$.

17. The elasticity of demand when Marginal Revenue is zero, is

   a) 1

   b) 2

   c) -5

   d) 0.

18. The marginal productivity of labour (L) for the production function $P = 15k - L^2 + 2kL$ when $L = 3$ and $k = 4$ is

     a)   21   b)   12

     c)   2   d)   3.

19. $\int\limits_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx =$

     a)   0   b) -1

     c)   1   d) $\dfrac{\pi}{2}$.

20. The area of the region bounded by $y = x + 1$, the x-axis and the line $x = 0$ and $x = 1$ is

     a) $\dfrac{1}{2}$ b)   2

     c) $\dfrac{3}{2}$ d)   1.

21. The marginal revenue of a firm is $MR = 15 - 8x$. Then the revenue function is

     a) $15x - 4x^2 + k$ b) $\dfrac{15}{x} - 8$

     c) -8 d) $15x - 8$.

22. The differential equation of straight lines passing through the origin is

     a) $x\dfrac{dy}{dx} = y$ b) $\dfrac{dy}{dx} = \dfrac{x}{y}$

     c) $\dfrac{dy}{dx} = 0$ d) $x\dfrac{dy}{dx} = \dfrac{1}{y}$.

23. The solution of $x\,dx + y\,dy = 0$ is

    a) $x^2 + y^2 = c$ b) $\dfrac{x}{y} = c$

    c) $x^2 - y^2 = c$ d) $xy = c.$

24. The integrating factor of $x\dfrac{dy}{dx} - y = e^x$ is

    a) $\log x$ b) $e^{\frac{-1}{x}}$

    c) $\dfrac{1}{x}$ d) $\dfrac{-1}{x}.$

25. The particular integral of the differential equation $\dfrac{d^2 y}{dx^2} - 5\dfrac{dy}{dx} + 6y = e^{5x}$ is

    a) $\dfrac{e^{5x}}{6}$ b) $\dfrac{x\,e^{5x}}{2!}$

    c) $6\,e^{5x}$ d) $\dfrac{e^{5x}}{25}.$

26. $E^2 f(x) =$

    a) $f(x+h)$ b) $f(x+2h)$

    c) $f(2h)$ d) $f(2x).$

27. When $h = 1$, $\Delta(x^2) =$

    a) $2x$ b) $2x - 1$

    c) $2x + 1$ d) $1.$

28. The normal distribution curve is

a) bimodal

b) unimodal

c) skewed

d) none of these.

29. If X is a Poisson variate with $P(X = 1) = P(X = 2)$ the mean of the Poisson variate is equal to

a) 1

b) 2

c) –2

d) 3.

30. If the probability density function of a variable X is defined as $f(x) = Cx(2 - x)$, $0 < x < 2$ then value of C is

a) $\dfrac{4}{3}$

b) $\dfrac{6}{4}$

c) $\dfrac{3}{4}$

d) $\dfrac{3}{5}$.

31. Chance variation in the manufactured product is

a) controllable

b) not controllable

c) both (a) & (b)

d) none of these.

32. The standard error of the sample mean is

a) Type I error

b) Type II error

c) Standard deviation of the sampling distribution of the mean

d) Variance of the sampling distribution of the mean.

[Turn over

33. If a random sample of size 64 is taken from a population whose standard deviation is equal to 32, then the standard error of the mean is

   a)   0·5

   b)   2

   c)   4

   d)   32.

34. The number of ways in which one can select 2 customers out of 10 customers is

   a)   90

   b)   60

   c)   45

   d)   50.

35. Which of the following statements is true ?

   a)   Point estimate gives a range of values.

   b)   Sampling is done only to estimate a statistic.

   c)   Sampling is done to estimate the population parameter.

   d)   Sampling is not possible for an infinite population.

36. The component of a time series attached to long term variation is termed as

   a)   cyclic variation

   b)   secular trend

   c)   irregular variation

   d)   all of these.

37. Cyclic variations in a time series are caused by

   a)   lockout in a factory

   b)   war in a country

   c)   floods in the states

   d)   none of these.

38. Most commonly used index number is

    a)   diffusion index number   b)   price index number

    c)   value index number   d)   none of these.

39. Control charts in statistical quality consist of

    a)   three control lines

    b)   upper and lower control limits

    c)   the level of process

    d)   all of these.

40. In a regression line of Y on X, the variable X is known as

    a)   independent variable   b)   dependent variable

    c)   both (a) and (b)   d)   none of these.

SECTION – B

N. B. :   Answer any *ten* out of *fifteen* questions given. $10 \times 6 = 60$

41. Given $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$. Verify that Adj $(AB) = (\text{Adj } B)(\text{Adj } A)$.

42. Solve using matrices the equations $2x - y = 3$, $5x + y = 4$.

43. Find the equation of the ellipse whose foci are $(4, 0)$ and $(-4, 0)$ and $e = \dfrac{1}{3}$.

[Turn over

44. The demand for a given commodity is $q = -60p + 480$ ($0 < p < 7$), where p is the price. Find the elasticity of demand and marginal revenue when $p = 6$.

45. A metal cylinder when heated, expands in such a way that its radius r increases at the rate of 0·2 cm per minute and its height h increases at a rate of 0·15 cm per minute retaining its shape. Determine the rate of change of volume of the cylinder when its radius is 10 cm and its height is 25 cm.

46. Calculate the EOQ in units and total variable cost for the following items assuming an ordering cost of Rs. 5 and a holding cost of 10%.

    Annual demand = 392 units.

    One unit price = Rs. 8·60

47. For the marginal cost function $MC = 5 - 6x + 3x^2$, x is the output. If the cost of producing 10 items is Rs. 850, find the total cost and average cost functions.

48. Solve : $\left(1 - e^x\right) \sec^2 y \, dy + 3e^x \tan y \, dx = 0$.

49. Solve : $\dfrac{dy}{dx} + y \cos x = \dfrac{1}{2} \sin 2x$.

50. If $f(0) = 5$, $f(1) = 6$, $f(3) = 50$ and $f(4) = 105$. find $f(2)$ by Lagrange's formula.

51. Fit a straight line $y = ax + b$ to the following data by the method of least squares :

x :	0	1	3	6	8
y :	1	3	2	5	4

52. It is stated that 2% of razor blades supplied by a manufacturer are defective. A random sample of 200 blades is drawn from a lot. Find the probability that 3 or more blades are defective $\left(e^{-4} = 0.01832\right)$.

53. Out of 1000 TV viewers, 320 watched a particular programme. Find 95% confidence limits for TV viewers who watched this programme.

54. Find trend values to the following data by the method of semi-average :

Year :	1980	1981	1982	1983	1984	1985	1986
Sales :	102	105	114	110	108	116	112

55. Calculate Fisher's Index numbers from the following data :

Commodity	Price		Quantity	
	Base year	Current year	Base year	Current year
A	6	10	40	50
B	2	2	90	120
C	4	6	70	60
D	10	12	40	25

SECTION – C

N. B. : Answer any *ten* questions. $10 \times 10 = 100$

56. Show that the equations

$2x - y + z = 7$

$3x + y - 5z = 13$

$x + y + z = 0$

are consistent and have unique solution.

57. The data below are about an economy of two industries P and Q. The values are

in millions of rupees.

Producer	User		Final Demand	Total Output
	P	Q		
P	14	6	8	28
Q	7	18	11	36

Determine the outputs if the final demand changes to 20 for P and 30 for Q.

58. Find the equation to the asymptotes of the hyperbola

$3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

59. At what points on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel

to (i) x-axis (ii) y-axis ?

60. Investigate the maxima and minima of the function $2x^3 - 9x^2 + 12x + 15$?

[Turn over

61. The demand for a commodity A is $q_1 = 240 - p_1^2 + 6p_2 - p_1 p_2$.

Find the partial elasticity $\dfrac{Eq_1}{Ep_1}$ and $\dfrac{Eq_1}{Ep_2}$ when $p_1 = 5$ and $p_2 = 4$.

62. Evaluate $\displaystyle\int_0^{\pi} x \sin^2 x \, dx$.

63. In a perfect competition the demand and supply curves of a commodity are given

by $P_d = 40 - x^2$ and $P_s = 3x^2 + 8x + 8$. Find the consumer's surplus and

producer's surplus at the market equilibrium price.

64. Solve : $\left(D^2 - 13D + 12\right) y = e^{-2x} + 5e^x$.

65. From the following data calculate the value of $e^{1.75}$.

x :	1·7	1·8	1·9	2·0	2·1
e^x :	5·474	6·050	6·686	7·389	8·166

66. A random variable X has the following probability distribution :

Values of X

x :	0	1	2	3	4	5	6	7	8
$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i)   Determine the value of a

ii)   Find $P(X < 3)$, $P(X > 3)$ and $P(0 < X < 5)$.

[Turn over

67. In a sample of 1000 candidates the mean of certain test is 45 and standard deviation is 15. Assuming the normality of the distribution, find the following :

   i)   How many candidates score between 40 and 60 ?

   ii)   How many candidates score above 50 ?

   iii) How many candidates score below 30 ?

   Use :

Z :	0·33	1
Area :	0·1293	0·3413

68. A sample of five measurements of the diameter of a sphere were recorded by a scientist as 6·33, 6·37, 6·36, 6·32 and 6·37 mm.

   Determine the point estimate of

   i)   mean

   ii)   variance.

69. Solve the following, using graphical method :

   Minimize $Z = 3x_1 + 2x_2$

   subject to constraints $5x_1 + x_2 \geq 10$

   $2x_1 + 2x_2 \geq 12$

   $x_1 + 4x_2 \geq 12$

   $x_1, x_2 \geq 0.$

70. From the data given below, calculate the Seasonal Indices :

Quarter	Year				
	1984	1985	1986	1987	1988
I	40	42	41	45	44
II	35	37	35	36	38
III	38	39	38	36	38
IV	40	38	40	41	42