Register Number

Part III — BUSINESS MATHEMATICS

(English Version)

Time Allowed : 3 Hours]   [Maximum Marks : 200

SECTION – A

N. B. :   i)   Answer *all* the 40 questions.

         ii)   Each question carries *one* mark.

         iii)   Choose and write the correct answer from the four choices given. $40 \times 1 = 40$

1.   If $AB = BA = \mid A \mid I$, then the matrix B is

    a)   the inverse of A b)   the transpose of A

    c)   the adjoint of A d) $2A$.

2.   The inverse of $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is

    a) $\begin{pmatrix} 0 & \frac{1}{2} \\ 2 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$

    c) $\begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$ d) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

[Turn over

3. The rank of an $n \times n$ matrix each of whose elements is 1 is

   a) 1

   b) 2

   c) n

   d) n^2.

4. If $T = \begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.7 & 0.3 \\ x & 0.8 \end{pmatrix} \end{array}$ is a transition probability matrix, then the value of x is

   a) 0·1

   b) 0·2

   c) 0·3

   d) 0·7.

5. The relation $R = \begin{array}{cc} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} a \\ b \end{array} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array}$ is

   a) Reflexive

   b) Symmetric

   c) Transitive

   d) Reflexive and symmetric.

6. Equation of the directrix of the parabola $x^2 = 4ay$ is

   a) $x + a = 0$

   b) $x - a = 0$

   c) $y + a = 0$

   d) $y - a = 0$.

7. The length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$ is

   a) $\dfrac{4}{3}$

   b) $\dfrac{8}{9}$

   c) $\dfrac{4}{9}$

   d) $\dfrac{8}{3}$

8. If 'a' is the length of the semi-transverse axis of rectangular hyperbola $xy = c^2$ then the value of c^2 is

   a) a^2

   b) $2a^2$

   c) $\dfrac{a^2}{2}$

   d) $\dfrac{a^2}{4}$

9. In an ellipse $e = \dfrac{3}{5}$, the length of semi-minor axis is 2. The length of major axis is

   a) 4

   b) 8

   c) 5

   d) 10.

10. The average fixed cost of the function $C = 2x^3 - 3x^2 + 4x + 8$ is

    a) $\dfrac{2}{x}$

    b) $\dfrac{4}{x}$

    c) $-\dfrac{3}{x}$

    d) $\dfrac{8}{x}$

11. Variable cost per unit is Rs. 40, fixed cost is Rs. 900 and unit selling price is Rs. 70. Then the profit equation is

    a) $P = 30x - 900$

    b) $P = 15x - 70$

    c) $P = 40x - 900$

    d) $P = 70x + 3600$.

12. The slope of the tangent at $(2, 8)$ on the curve $y = x^3$ is

    a) 3

    b) 6

    c) 12

    d) 8.

13. If $y = 2x^2 + 3x$, the instantaneous rate of change of y at $x = 4$ is

a) 16

b) 19

c) 30

d) 4.

14. Given the demand equation $p = -x + 10$; ($0 \leq x \leq 10$) where p denotes the u selling price and x denotes the number of units demanded of some product. 7 marginal revenue at $x = 3$ units is

a) Rs. 5

b) Rs. 10

c) Rs. 4

d) Rs. 30.

15. The cost function $y = 40 - 4x + x^2$ is minimum when

a) $x = 2$

b) $x = -2$

c) $x = 4$

d) $x = -4$.

16. The curve $y = 4 - 2x - x^2$ is

a) concave upward

b) concave downward

c) straight line

d) none of these.

17. The elasticity of demand when marginal revenue is zero, is

a) 1

b) 2

c) -5

d) 0.

18. The stationary value of x for the function $f(x) = 3(x-1)(x-2)$ is

   a)   3   b) $\dfrac{3}{2}$

   c) $\dfrac{2}{3}$ d) $-\dfrac{3}{2}$

19. $\displaystyle\int_{-3}^{3} x \, dx$ is

   a)   0   b)   2

   c)   1   d)   – 1.

20. If the marginal cost function $MC = 2 - 4x$, then the cost function is

   a) $2x - 2x^2 + k$ b) $2 - 4x^2$

   c) $\dfrac{2}{x} - 4$ d) $2x - 4x^2$.

21. The area bounded by $y = x$, y-axis and $y = 1$ is

   a)   1   b) $\dfrac{1}{2}$

   c)   log 2   d)   2.

22. The degree and order of the differential equation $\dfrac{d^2y}{dx^2} - 6\sqrt{\dfrac{dy}{dx}} = 0$ are

   a)   2 and 1   b)   1 and 2

   c)   2 and 2   d)   1 and 1.

23. The solution of $x\,dy + y\,dx = 0$ is

    a) $x + y = c$

    b) $x^2 + y^2 = c$

    c) $xy = c$

    d) $y = cx.$

24. The integrating factor of $x\dfrac{dy}{dx} - y = e^x$ is

    a) $\log x$

    b) $e^{-\frac{1}{x}}$

    c) $\dfrac{1}{x}$

    d) $-\dfrac{1}{x}$

25. The particular integral of the differential equation $\dfrac{d^2 y}{dx^2} - 5\dfrac{dy}{dx} + 6y = e^{5x}$

    a) $\dfrac{e^{5x}}{6}$

    b) $\dfrac{x\,e^{5x}}{2!}$

    c) $6\,e^{5x}$

    d) $\dfrac{e^{5x}}{25}$

26. In a line of best fit $y = 5{\cdot}8\,(x - 1994) + 41{\cdot}6$ the value of y when $x = 1997$

    a)   60

    b)   50

    c)   54

    d)   59.

27. $E^2 f(x) =$

    a) $f(x + h)$

    b) $f(x + 2h)$

    c) $f(2h)$

    d) $f(2x).$

28. The mean and variance of a binomal distribution are

    a) np, npq b) pq, npq

    c) np, $\sqrt{npq}$ d) np, nq.

29. If X is a Poisson variate with $P(X = 1) = P(X = 2)$, the mean of the Poisson variate is equal to

    a) 1   b) 2

    c) -2 d) 3.

30. The random variables X and Y are independent if

    a) $E(XY) = 1$ b) $E(XY) = 0$

    c) $E(XY) = E(X)E(Y)$ d) $E(X+Y) = E(X) + E(Y)$.

31. If $X \sim N(\mu, \sigma^2)$, the maximum probability at the points of inflexion of normal distribution is

    a) $\dfrac{1}{\sqrt{2\pi}} e^{\frac{1}{2}}$ b) $\dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}$

    c) $\dfrac{1}{\sigma\sqrt{2\pi}}$ d) $\dfrac{1}{\sqrt{2\pi}}$.

32. The Z-value that is used to establish a 95% confidence interval for the estimation of a population parameter is

    a) $1 \cdot 28$ b) $1 \cdot 65$

    c) $1 \cdot 96$ d) $2 \cdot 58$.

33. The standard error of the sample mean is

    a) Type I error

    b) Type II error

    c) standard deviation of the sampling distribution of the mean

    d) variance of the sampling distribution of the mean.

34. If a random sample of size 64 is taken from a population whose standard deviation is equal to 32, then the standard error of the mean is

    a) 0·5   b) 2

    c) 4   d) 32.

35. Which of the following statements is true ?

    a) Point estimates give a range of values

    b) Sampling is done only to estimate a statistic

    c) Sampling is done to estimate the population parameter

    d) Sampling is not possible for an infinite population.

36. A time series is a set of data recorded

    a) periodically   b) at equal time intervals

    c) at successive points of time   d) all of these.

37. The component of a time series which is attached to short term fluctuations is

    a) seasonal variation
    b) cyclic variation

    c) irregular variation
    d) all of these.

38. The weights used in Paasche's formula belong to

    a) the base period
    b) the current period

    c) to any arbitrary chosen period
    d) none of these.

39. The range of correlation coefficient is

    a) 0 to ∞
    b) – ∞ to ∞

    c) – 1 to 1
    d) none of these.

40. Scatter diagram of the variate values (X, Y) gives the idea about

    a) functional relationship
    b) regression model

    c) distribution of errors
    d) none of these.

SECTION - B

N. B. : i) Answer any *ten* out of *fifteen* questions given.

   ii) Each question carries *six* marks. $10 \times 6 = 60$

41. Show that the equations $x - 3y + 4z = 3$, $2x - 5y + 7z = 6$, $3x - 8y + 11z = 1$

   are inconsistent.

42. Verify $(AB)^{-1} = B^{-1} A^{-1}$, when $A = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -6 & 0 \\ 0 & 9 \end{pmatrix}$.

43. Find the focus, latus rectum, vertex, directrix of the parabola $x^2 - 3y + 3 = 0$.

44. The total cost C of producing x units is

$C = 0.00004x^3 - 0.002x^2 + 3x + 10,000$. Find the marginal cost of 1000 units output.

45. Find the equation of the tangent and normal to the demand curve $y = 36 - x^2$ when $y = 11$.

46. A certain manufacturing concern has the total cost function $C = \frac{1}{5}x^2 - 6x + 100$.

When will the total cost be minimum ?

47. If the marginal revenue for a commodity is $MR = 9 - 6x^2 + 2x$, find the total revenue and demand function.

48. Solve $\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$.

49. Solve $\frac{dy}{dx} + \frac{2xy}{1 + x^2} = \frac{1}{(1 + x^2)^2}$ given that $y = 0$ when $x = 1$.

50. Apply Lagrange's formula to find y when $x = 5$, given that

x :	1	2	3	4	7
y :	2	4	8	16	128

51. If $y_{75} = 2459$, $y_{80} = 2018$, $y_{85} = 1180$ and $y_{90} = 402$, find y_{82}.

(Use **Gregory – Newton's** forward formula)

52. Find the probability that at most 5 defective fuses will be found in a box 200 fuses if experience shows that 2% of such fuses are defective.

$(e^{-4} = 0.0183)$.

53. A random sample of 500 apples was taken from large consignment and 45 them were found to be bad. Find the limits at which the bad apples lie at 99 confidence level.

54. Find the trend values to the following data by the method of semi-averages :

Year :	1980	1981	1982	1983	1984	1985	1986
Sales :	102	105	114	110	108	116	112

55. Solve the following, using graphical method :

Maximize $Z = 45x_1 + 80x_2$

Subject to the constraints

$$5x_1 + 20x_2 \leq 400$$

$$10x_1 + 15x_2 \leq 450$$

$$x_1, x_2 \geq 0.$$

SECTION – C

N. B. : i) Answer any *ten* questions out of *fifteen* questions given.

ii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Solve by Cramer's rule :

$2x + 2y - z = 1, x + y - z = 0, 3x + 2y - 3z = 1.$

57. In an economy of two industries P and Q the following table gives the supply and demand positions in crores of rupees :

Producer	User		Final demand	Total output
	P	Q		
P	50	75	75	200
Q	100	50	50	200

Find the outputs when the final demand changes to 300 for P and 600 for Q.

58. Find the equations of the asymptotes of the hyperbola

$$2x^2 + 5xy + 2y^2 - 11x - 7y - 4 = 0.$$

59. If AR and MR denote the average and marginal revenue at any output level, show that the elasticity of demand is equal to $\dfrac{AR}{AR - MR}$. Verify this for the linear demand law $p = a + bx$. where p is price and x is the quantity.

60. The annual demand for an item is 3200 units. The unit cost is Rs. 6 and inventory carrying charges 25% per annum. If the cost of one procurement is Rs. 150, determine (i) Economic order quantity (ii) Time between two consecutive orders.

61. The demand for a commodity A is $q_1 = 240 - p_1^2 + 6p_2 - p_1 p_2$. Find partial elasticities $\dfrac{Eq_1}{Ep_1}$ and $\dfrac{Eq_1}{Ep_2}$ when $p_1 = 5$ and $p_2 = 4$.

62. Evaluate $\displaystyle\int_{\pi/6}^{\pi/3} \dfrac{dx}{1 + \sqrt{\cot x}}$.

63. The demand and supply function for a commodity are given by $P_d = 15 - x$ a $P_s = 0.3x + 2$. Find the consumers' surplus and producers', surplus at market equilibrium price.

64. Suppose that $Q_d = 30 - 5P + 2\dfrac{dP}{dt} + \dfrac{d^2P}{dt^2}$ and $Q_s = 6 + 3P$, where P deno price. Find the equilibrium price for market clearance.

65. Fit a straight line $y = ax + b$ to the following data by the method of le squares :

x :	0	1	3	6	8
y :	1	3	2	5	4

66. The I.Q. (intelligence quotient) of a group of 1000 children has mean 96 and standard deviation 12. Assuming the distribution as normal, find approximat the number of children having I.Q.

i)   less than 72

ii)   between 80 and 120.

    [Given $P(0 \le Z \le 2) = 0.4772$, $P(0 \le Z \le 1.33) = 0.4082$]

67. In a continous distribution, whose probability density function is given by

$f(x) = \frac{3}{4} x (2 - x)$, $0 \le x \le 2$, show that the arithmetic mean of the distribution

is 1 and the variance is $\frac{1}{5}$.

68. The mean life time of 50 electric bulbs produced by a manufacturing company is

estimated to be 825 hours with a standard deviation of 110 hours. If μ is the

mean life time of all the bulbs produced by the company, test the hypothesis

that $\mu = 900$ hours at 5% level of significance.

69. From the following data calculate the price index number by (a) Laspeyre's

method, (b) Paasche's method and (c) Fisher's method :

	Base year		Current year	
Commodity	Price	Quantity	Price	Quantity
A	5	25	6	30
B	10	5	15	4
C	3	40	2	50
D	6	30	8	35

70. The following data shows the value of sample mean $\bar{X}$ and the range R for ten samples of size 6 each. Calculate the values for central line and control limits for mean chart and range chart and determine whether the process is in control :

Sample no.	1	2	3	4	5	6
Mean $\bar{X}$	681	586	651	641	680	639
Range R	118	167	134	171	490	200

Sample no.	7	8	9	10
Mean $\bar{X}$	665	604	569	629
Range R	236	188	309	257

(Given for $n = 6$, $A_2 = 0.483$, $D_3 = 0$, $D_4 = 2.004$).