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Part III — BUSINESS MATHEMATICS

(English Version)

Time Allowed : 3 Hours]

[Maximum Marks : 200

SECTION - A

- N. B. :
- Answer *all* the 40 questions.
 - Each question carries *one* mark.
 - Choose and write the correct answer from the four choices given.
- 40 × 1 = 40

1. If $A = \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$ then A^{-1} is

a) $\begin{pmatrix} -0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}$

b) $\begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$

c) $\begin{pmatrix} 0.8 & 0.6 \\ 0.6 & 0.8 \end{pmatrix}$

d) $\begin{pmatrix} 0.2 & 0.4 \\ -0.4 & 0.2 \end{pmatrix}$.

2. The rank of $n \times n$ matrix each of whose element is 2, is

a) 1

b) 2

c) n

d) n^2 .

[Turn over

3. If $T = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{pmatrix} 0.7 & 0.3 \\ x & 0.8 \end{pmatrix} \end{matrix}$ is a transition probability matrix, then the value of x is

- a) 0.3
- b) 0.2
- c) 0.8
- d) 0.7.

4. If $|A| = 0$, then $|\text{adj } A|$ is

- a) 0
- b) 1
- c) -1
- d) ± 1 .

5. A system of linear homogeneous equations has at least

- a) one solution
- b) two solutions
- c) three solutions
- d) four solutions.

6. Equation of the directrix of $x^2 = 4ay$ is

- a) $x + a = 0$
- b) $x - a = 0$
- c) $y + a = 0$
- d) $y - a = 0$.

7. The semi-major and semi-minor axes of $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is

- a) (4, 5)
- b) (8, 10)
- c) (5, 4)
- d) (10, 8).

8. Eccentricity of the rectangular hyperbola is

a) 2

b) $\frac{1}{2}$

c) $\sqrt{2}$

d) $\frac{1}{\sqrt{2}}$

9. The difference between the focal distances of any point on the hyperbola is equal to length of its

a) transverse axis

b) semi-transverse axis

c) conjugate axis

d) semi-conjugate axis.

10. The average fixed cost of the function $C = 2x^3 - 3x^2 + 4x + 8$ is

a) $\frac{2}{x}$

b) $\frac{4}{x}$

c) $-\frac{3}{x}$

d) $\frac{8}{x}$

11. If the rate of change of y with respect to x is 6 and x is changing at 4 units/sec, then the rate of change of y per second is

a) 24 units/sec

b) 10 units/sec

c) 2 units/sec

d) 22 units/sec.

12. For the curve $y = 1 + ax - x^2$ the tangent at $(1, -2)$ is parallel to X-axis. The value of a is

a) -2

b) 2

c) 1

d) -1.

[Turn over

13. The slope of the tangent to the curve $y = x^2 - \log x$ at $x = 2$ is

a) $\frac{7}{2}$

b) $\frac{2}{7}$

c) $-\frac{7}{2}$

d) $-\frac{2}{7}$

14. Variable cost per unit is Rs. 40, fixed cost is Rs. 900 and unit selling price is Rs. 70. Then the profit equation is

a) $P = 30x - 900$

b) $P = 15x - 70$

c) $P = 40x - 900$

d) $P = 70x + 3600$

15. The curve $y = 4 - 2x - x^2$ is

a) concave upward

b) concave downward

c) straight line

d) none of these.

16. If $u = e^{x^2 + y^2}$ then $\frac{\partial u}{\partial x}$ is equal to

a) $y^2 u$

b) $x^2 u$

c) $2xu$

d) $2yu$

17. The elasticity of demand when marginal revenue is zero, is

a) 1

b) 2

c) -5

d) 0.

18. The cost function $y = 40 - 4x + x^2$ is minimum when

a) $x = 2$

b) $x = -2$

c) $x = 4$

d) $x = -4.$

19. The area bounded by the curve $y = e^x$, the x -axis and the lines $x = 0$ and $x = 2$ is

a) $e^2 - 1$

b) $e^2 + 1$

c) e^2

d) $e^2 - 2.$

20. The marginal revenue of a function is $MR = 15 - 8x$. Then the revenue function is

a) $15x - 4x^2 + k$

b) $\frac{15}{x} - 8$

c) -8

d) $15x - 8.$

21. The producers' surplus for the supply function $p = g(x)$ for the quantity x_0 and price p_0 is

a) $\int_0^{x_0} g(x) dx - p_0 x_0$

b) $p_0 x_0 - \int_0^{x_0} g(x) dx$

c) $\int_0^{x_0} g(x) dx$

d) $\int_0^{p_0} g(x) dx.$

[Turn over

22. The order and degree of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - 3\frac{d^3y}{dx^3} + 7\frac{d^2y}{dx^2} + \frac{dy}{dx} = x + \log x \text{ are}$$

- | | |
|------------|-------------|
| a) 1 and 3 | b) 3 and 1 |
| c) 2 and 3 | d) 3 and 2. |

23. The solution of $x dx + y dy = 0$ is

- | | |
|--------------------|----------------------|
| a) $x^2 + y^2 = c$ | b) $\frac{x}{y} = c$ |
| c) $x^2 - y^2 = c$ | d) $xy = c.$ |

24. The integrating factor of $\frac{dy}{dx} + \frac{2y}{x} = x^3$ is

- | | |
|-------------------|--------------|
| a) $2 \log x$ | b) e^{x^2} |
| c) $3 \log (x^2)$ | d) $x^2.$ |

25. The complementary function of the differential equation $(D^2 - 2D + 1)y = e^{2x}$ is

- | | |
|---------------------|-------------------|
| a) $Ae^x + Be^{-x}$ | b) $A + Be^x$ |
| c) $(Ax + B)e^x$ | d) $A + Be^{-x}.$ |

26. $E^2 f(x) =$

- | | |
|---------------|----------------|
| a) $f(x + h)$ | b) $f(x + 2h)$ |
| c) $f(2h)$ | d) $f(2x).$ |

27. The normal equations of fitting a straight line $y = ax + b$ are $10a + 5b = 15$ and $30a + 10b = 43$. The slope of the line of best fit is

- a) 1.2 b) 1.3
c) 13 d) 12.

28. If the probability density function of variable X is defined as $f(x) = Cx(2-x)$, $0 < x < 2$, then the value of C is

- a) $\frac{4}{3}$ b) $\frac{3}{2}$
c) $\frac{3}{4}$ d) $\frac{3}{5}$.

29. If $X \sim N(\mu, \sigma^2)$, the points of inflection of normal distribution curve are

- a) $\pm \mu$ b) $\mu \pm \sigma$
c) $\sigma \pm \mu$ d) $\mu \pm 2\sigma$.

30. If a random variable X has the following probability distribution :

$x :$	- 1	- 2	1	2
$P(x) :$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

then the expected value of x is

- a) $\frac{3}{2}$ b) $\frac{1}{6}$
c) $\frac{1}{2}$ d) $\frac{1}{3}$.

[Turn over

31. If X is a Poisson variate with $P (X = 1) = P (X = 2)$, the mean of the Poisson variate is equal to
- a) 1
 - b) 2
 - c) - 2
 - d) 3.
32. If a random sample of size 64 is taken from a population whose standard deviation is equal to 32, then the standard error of the mean is
- a) 0.5
 - b) 2
 - c) 4
 - d) 32.
33. The Z -value that is used to establish a 95% confidence interval for the estimation of a population parameter is
- a) 1.28
 - b) 1.65
 - c) 1.96
 - d) 2.58.
34. Which of the following statements is true ?
- a) Point estimates give a range of values.
 - b) Sampling is done only to estimate a statistic.
 - c) Sampling is done to estimate the population parameter.
 - d) Sampling is not possible for an infinite population.

35. The number of ways in which one can select 2 customers out of 10 customers is
- a) 90 b) 60
c) 45 d) 50.
36. The component of a time series attached to long term variation is termed as
- a) cyclic variation b) secular trend
c) irregular variation d) all of these.
37. Index number is a
- a) measure of relative changes b) a special type of an average
c) a percentage relative d) all of these.
38. The weights used in Paasche's formula belong to
- a) the base period b) the current period
c) to any arbitrary chosen period d) none of these.
39. The range of correlation coefficient is
- a) 0 to ∞ b) $-\infty$ to ∞
c) - 1 to 1 d) none of these.
40. The lines of regression intersect at the point
- a) (X, Y) b) (\bar{X} , \bar{Y})
c) (0, 0) d) none of these.

[Turn over

SECTION - B

N. B. : i) Answer any *ten* out of *fifteen* questions given.

ii) Each question carries *six* marks.

$10 \times 6 = 60$

41. Find the inverse of $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ if it exists.

42. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -2 & -4 \end{bmatrix}$.

43. Find the equation of the ellipse whose focus is $(1, -2)$, directrix is

$$3x - 2y + 1 = 0 \text{ and eccentricity } e = \frac{1}{\sqrt{2}}.$$

44. Find the equilibrium price and equilibrium quantity for the following demand and supply function $q_d = 4 - 0.05p$ and $q_s = 0.8 + 0.11p$.

45. A metal cylinder when heated, expands in such a way that its radius r , increases at the rate of 0.2 cm per minute and its height h increases at a rate of 0.15 cm per minute retaining its shape. Determine the rate of change of volume of the cylinder when its radius is 10 cm and its height is 25 cm.

46. The demand for commodity A is $q_1 = 240 - p_1^2 + 6p_2 - p_1p_2$. Find the partial elasticity $\frac{Eq_1}{Ep_2}$ when $p_1 = 5$ and $p_2 = 4$.

47. The marginal cost function of manufacturing x units of a commodity is $3x^2 - 2x + 8$. If there is no fixed cost, find the total cost and average cost function.

48. Solve : $(1 - x^2) \frac{dy}{dx} - xy = 1$.

49. Solve : $2 \frac{dy}{dx} = \frac{y}{x} - \frac{y^2}{x^2}$.

50. From the following data, find $f(3)$:

$x :$	1	2	3	4	5
$f(x) :$	2	5	-	14	32

51. Using Lagrange's formula find the value of y when $x = 42$ from the following data :

$x :$	40	50	60	70
$y :$	31	73	124	159

52. In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution.

53. A random sample of 500 apples was taken from large consignment and 45 of them were found to be bad. Find the limits at which the bad apples lie at 99% confidence level.

[Turn over

54. Solve graphically, maximize $Z = 3x_1 + 4x_2$ subject to the constraints

$$2x_1 + x_2 \leq 40, \quad 2x_1 + 5x_2 \leq 180, \quad x_1, x_2 \geq 0.$$

55. Find the trend values to the following data by the method of semi-average :

Year :	1980	1981	1982	1983	1984	1985	1986
Sales :	102	105	114	110	108	116	112

SECTION - C

N. B. : i) Answer any *ten* questions out of *fifteen* questions given.

ii) Each question carries *ten* marks.

10 × 10 = 100

56. A salesman has the following record of sales during three months for three items A, B and C which have different rates of commission :

Months	Sale of Units			Total Commission drawn (in Rs.)
	A	B	C	
January	90	100	20	800
February	130	50	40	900
March	60	100	30	850

Find out the rates of commission on the items A, B and C. Solve by Cramer's Rule.

57. In an economy of two industries of P and Q the following table gives the supply and demand position in millions of rupees :

<i>Producer</i>	<i>User</i>		<i>Final demand</i>	<i>Total output</i>
	<i>P</i>	<i>Q</i>		
P	16	20	4	40
Q	8	40	32	80

Find the output when the final demand changes to 18 for P and 44 for Q .

58. Find the centre, eccentricity, foci and latus rectum of the hyperbola

$$9x^2 - 16y^2 - 18x - 64y - 199 = 0.$$

59. Find the equation of the tangent and normal to the supply curve $y = x^2 + x + 2$ when $x = 6$.

60. Find the maximum and minimum values of the function $x^3 - 6x^2 + 9x + 15$.

61. If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, using Euler's theorem prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0.$$

62. The demand and supply laws under pure competition are given by $P_d = 23 - x^2$ and $P_s = 2x^2 - 4$. Find the Consumers' surplus and Producers' surplus at the market equilibrium price.

[Turn over

63. Find the area of one loop of the curve $y^2 = x^2(4 - x^2)$ between $x = 0$ and $x = 2$.

64. Solve : $(D^2 - 13D + 12)y = e^{-2x} + 5e^x$.

65. Fit a straight line to the following data :

x :	4	8	12	16	20	24
y :	7	9	13	17	21	25

66. Let x be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{2} & \text{for } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) $E(X)$

(ii) $E(X^2)$

(iii) $Var(X)$.

67. A large number of measurements is normally distributed with a mean of $65.5''$ and standard deviation of $6.2''$. Find the percentage of measurements that fall between $54.8''$ and $68.8''$.

Use :

z :	0.53	1.73
Area :	0.2019	0.4582

68. A company markets car tyres. Their lives are normally distributed with a mean 50000 kms and standard deviation of 2000 kilometres. The company expects rise in sales subject to the new method of manufacturing. A test sample of 64 new tyres has a mean life of 51250 kms. Can you conclude that the sample mean differs significantly from the population mean ? (Test at 5% level)

69. Compute (a) Laspeyre's, (b) Paasche's and (c) Fisher's method of index number for 1990 from the following :

Commodity	Price		Quantity	
	1980	1990	1980	1990
A	2	4	8	6
B	5	6	10	5
C	4	5	14	10
D	2	2	19	13

70. Obtain the coefficient of correlation from the following data :

X :	10	12	18	24	23	27
Y :	13	18	12	25	30	10
