Sample Questions

RE-1

1. (a) Find

$$\int_0^{\pi} \left| \frac{1}{2} + \cos x \right| dx$$

   (b) Let two differentiable functions $f(x)$ and $g(x)$ be defined on the same domain. Further, $f(x) \geq 0$, $g(x) \leq 0$, $f(x)$ is strictly decreasing and $g(x)$ is strictly increasing. What can you say about the monotonic property of $h(x)$, where $h(x) = f(x)g(x)$? Justify your answer.

2. (a) Find the values of x for which the inequality

$$\left|x^2 - 7x + 12\right| > x^2 - 7x + 12$$

   holds.

   (b) Find the maximum value of $\frac{35^n}{n!}$ for any positive integer n.

3. (a) $P(x)$ is a quadratic polynomial such that $P(1) = -P(2)$. If -1 is a root of the equation, what is the other root?

   (b) Suppose the upper contour sets of f,$v(y) = \{x : f(x) \geq y\}$, where $y$is a scalar, $x = (x_1, x_2....x_n) \in R^n_+$and $f : R^n_+ \rightarrow R^1_+$is non-decreasing in individual arguments are convex sets for all $y \in R$. Then show that $f$is quasi concave, that is, show that for x^o,$x^{oo}$with $x^o \geq x^{oo}$,

$$f\left(\lambda x^o + (1-\lambda)\, x^{oo}\right) \geq f\left(x^{oo}\right) \quad \text{for any } \lambda \in [0,1].$$

4. Consider the discrete probability distribution defined by the probability function

$$f(x) = (1-\theta)\theta^x, \quad x = 0, 1, 2, ...$$
$$= 0 \qquad \text{otherwise}$$

   where $0 < \theta < 1$.

   (a) Write down the joint probability function for a random sample of size n from that population.

Sample Questions

RE-II

1. Consider a multi-plant monopolist with two production units.

   (a) The cost function of the i-th plant is $1 + q_i$, if $q_i > 0$ and zero otherwise. Let the demand function facing the monopolist be $q = 10 - p$. Solve for the optimal monopoly outcome.

   (b) How would your answer change if the cost function of the i-th plant is $1 + q_i$, $\forall q_i \geq 0$? Explain.

   (c) How would your answer change if the cost function of the i-th plant is $1 + q_i^2$, if $q_i > 0$ and zero otherwise? Explain.

   [9+8+8]

2. Consider a two period, world, exchange-endowment economy, with no governments and a single good. There are two countries, A and B. Country A has incomes y_1 and y_2 and consumption c_1 and c_2 in the two periods. Country B has incomes x_1 and x_2 and consumption d_1 and d_2 in the two periods. Suppose that people in country A maximize

$$ln(c_1) + \beta ln(c_2)$$

   and the utility function in B is similar.

   (a) Write out the budget constraint for each country. Also write the world market clearing conditions in each period.

   (b) If $y_1 = x_1 = 1$ and $y_2 = x_2 = 1$, then solve for c_1, c_2, d_1, d_2, and the trade balance in each country and time period, in a competitive equilibrium. Assume that $r = \frac{1}{\beta}$. Set $\beta = .9$.

   [12+13]

3. Consider two vertically related firms, A and B. Each is a monopolist. A produces good F which is used by B as input in his own production of good X. B's production function is given by

$$X = f(F) = F.$$

The demand function for X is given by

$$p_X = a - bX.$$

The marginal cost of monopolist A is constant and given by v. Monopolist B incurs a cost of c per unit produced in addition to the cost of the input from A. Show that the final output levels when the firms are integrated are twice the output levels when they are not integrated.

[25]

4. Does first differencing reduce autocorrelation? Compare the autocorrelation in the disturbance terms of the following models in answering this question:

$$y_t = \beta x_t + \varepsilon_t$$

with

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t$$

and

$$y_t - y_{t-1} = \beta(x_t - x_{t-1}) + (\varepsilon_t - \varepsilon_{t-1})$$

[25]

5. Suppose wage income (X_1) and property income (X_2) have a bivariate normal distribution with means (in thousands of rupees), $\mu_1 = 16$ and $\mu_2 = 10$ and variances $\sigma_1^2 = 16$ and $\sigma_2^2 = 25$ respectively, and covariance $\sigma_{12} = 2$. Let total income be given by $Y = X_1 + X_2$.

   (a) Calculate the expectation $E(Y)$ and the variance $V(Y)$ of Y.

   (b) Calculate $E(Y \mid X_1)$ and $V(Y \mid X_1)$

   (c) Predict the total income of a person whose wage income is 24 thousand rupees.

[10+10+5]

6. Suppose a farmer's output of rice (denoted by Y) is determined by the use of labor (X) and soil quality (C) in addition to random factors such as weather (U). A researcher wants to estimate the production function and

collects data on Y and X from N farmers. Soil quality is unobservable. The true model is given by

$$y_i = c_i + \beta x_i + u_i, \qquad 0 < \beta < 1, \quad i = 1, 2, ...N$$

where the lower case letters denote the logarithms of the upper case letters. In other words, the production function is Cobb-Douglas. Since C is not observed, the researcher regresses y on x and a constant, i.e.

$$y_i = \alpha + \tilde{\beta}\ x_i + e_i.$$

Compare $\tilde{\beta}$ and β. Are they equal? Explain.

[25]

7. There are N individuals located in the interval $[0, 1]$. The location of individual i is denoted by x_i, where $0 \leq x_i \leq 1$. A social planner has to decide the location of a swimming pool in $[0, 1]$. If the pool is built at y, where $y \in [0, 1]$), then individual i obtains utility $-|y - x_i|$.

   (a) Suppose the planner wishes to maximize the utility of the worst off individual by his choice of pool location. What location will he choose?

   (b) Suppose the planner wishes to maximize the sum of utilities of all individuals. What location will he choose?

   (c) Suppose that the planner has to decide on the location of a garbage dump instead of a swimming pool. All individuals want to be as far from the garbage dump as possible. Specifically, if the garbage dump is located at y, then i's utility is given by $|y - x_i|$. Suppose the planner wishes to maximize the sum of individual utilities. What location will he choose?

[9+8+8]

8. There are three candidates, A, B and C, and two voters, 1 and 2, in an election. Voter 1 gets a payoff of 1 if A is elected, 0.5 if B is elected and 0 if C is elected. Voter 2 gets a payoff of 1 if B is elected, 0.5 if A is elected and 0 if C is elected. The voting procedure is as follows. First voter 1 vetos (or eliminates) one of the three candidates. Voter 2 then vetos one of the two remaining candidates. The candidate who has not been vetoed by either 1 or 2 is elected.

(a) Write down an extensive form as well as a normal form for this game. What are the strategy sets for the two voters?

(b) Suppose voter 2 always vetos candidate A whenever 1 has not vetoed A and vetoes C otherwise. Voter 1 vetos A so that the outcome is B. Is this a Nash equilibrium of the game? If it is one, do you think it is likely to arise in actual play? Justify your answer carefully.

(c) Show that there is a Nash equilibrium of the game where voter 1 does not veto A.

[10+10+5]

9. Suppose we divide all households in an economy into two income classes: rich and poor. At time period 0, 90% of households are poor. In each subsequent period a household switches income classes with probability $\frac{1}{4}$ and remains in their income class with probability $\frac{3}{4}$.

(a) Examine whether the share of rich households in this economy converges.

(b) Would your answer to (a) change if all households started off poor? Explain.

[15+10]

10. Three friends, 1,2 and 3 take part in the following pizza-splitting game. In period 1, friend 1 is asked to split the pizza of size 1 into at most two parts (i.e. he is free to not split the pizza at all). Let the two resulting fractions of the pizza be p_1 and $1 - p_1$. In period 2, friend 2 is asked to make a single split on any one of the existing pizza parts (again, he is free to make no split at all). Once the two friends have made their splitting decisions, period 3 comes and friend 3 is asked to choose one of the possible many pizza parts on the table. This is followed by friend 2 choosing one of the remaining parts. Whatever is left is consumed by friend 1.

If you were friend 1 (and you and your friends all liked pizza), what would your choice of p_1 be?

[25]

(b) Calculate (in terms of θ) the probability of observing the random sample $\underline{x}$=(1,0,1,2,6).

5. When is the sum

$$r + \frac{r}{1+r} + \frac{r}{(1+r)^2} + \cdots$$

convergent? And what is the sum when it is convergent?

6. (a) For what values of a are the vectors $(0,1,a)$, $(a,1,0)$ and $(1,a,1)$ in R^3 linearly independent?

   (b) Find the rank of the matrix:

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 0 \\ 4 & 1 & 4 \end{bmatrix}$$

7. Solve $Max_{x,y,z} x^2 + y^2 + z^2$ subject to the constraints $4x + y + 3z \leq 10$ and $x, y, z \geq 0$.

8. How many ordered triples of strictly positive integers (x, y, z) are solutions to the equation $x + y + z = n$ where n is a fixed positive integer?

9. (a) Let $f(x)$ be a non-differentiable function and take $g(x)$=$f(x)^2$. Is it true that $g(x)$ is non-differentiable? Justify your answer.

   (b) In the game of *Parcheesi*, players take it in turns to roll two dice. At the start of the game, a player must bring a token out of base if she gets at least one five, or if the sum of the numbers on the two dice adds up to five. What is the probability that a player will get out of base on her first turn?