IIT-JEE 2004 Mains Questions & Solutions – Physics – Version 2

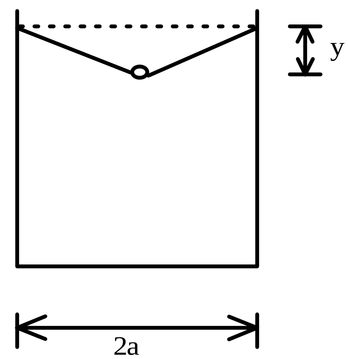
(The questions are based on memory)

Break-up of questions:

	Mechanics	Sound	Heat	Electricity & Magnetism	Optics	Modern Physics
Marks	**18 (30%)**	**0**	**10 (17%)**	**16 (26%)**	**10 (17%)**	**6 (10%)**

Remarks: Mechanics + Electricity & Magnetism constitute more than 50 % marks as expected. However, the paper was somewhat imbalanced as there was no question asked from Sound.

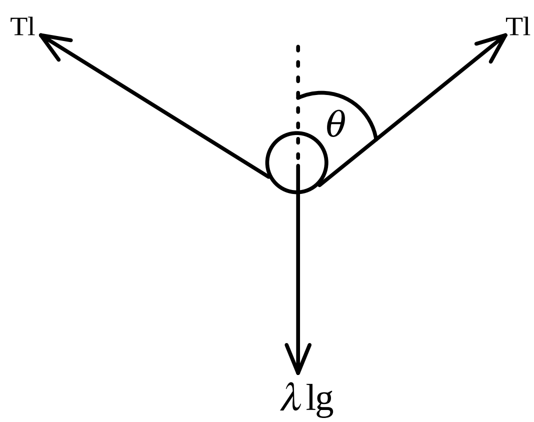
1. A container of width $2a$ is filled with a liquid. A thin wire of mass per unit length λ is gently placed over the liquid surface in the middle of the surface as shown in the figure. As a result, the liquid surface is depressed by a distance y ($y << a$). Determine the surface tension of the liquid.



[2]

Solution

Let T be the surface tension of the liquid. Drawing the FBD of the wire

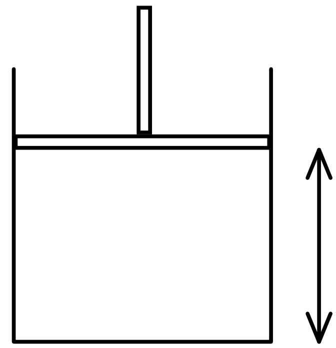


Considering the equilibrium of the wire in vertical direction, $2lT\cos\theta = \lambda lg$

$$\cos\theta \approx \frac{y}{a} \quad (y << a)$$

$$\therefore\ T = \frac{\lambda}{2\cos\theta g} \approx \frac{\lambda a}{2yg}$$

2. An isolated piston+cylinder arrangement shown contains a diatomic ideal gas at 300 K at equilibrium. The mass of the piston is negligible. The cross-sectional area of the cylinder is 1 m^2. Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brought back to its original height adiabatically, find the new equilibrium temperature of the gas. You can leave the answer in fraction.



[2]

Solution

At constant pressure (isobaric process),

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \Rightarrow \quad \frac{Ah}{300} = \frac{Ah'}{400} \quad \Rightarrow \quad h' = \frac{4}{3}h = \frac{4}{3}\text{ m}$$

For adiabatic process, $T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1}$

$$400\left(\frac{4}{3}\right)^{1.4-1} = T_3 (1)^{1.4-1} \qquad (\gamma = 1.4 \text{ for a diatomic gas})$$

$$\Rightarrow \quad T_3 = 400\left(\frac{4}{3}\right)^{0.4} \text{ K}$$

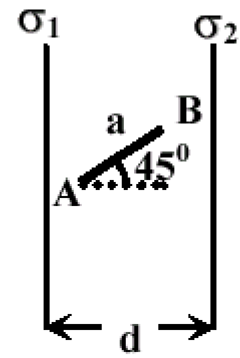
3. There are two large parallel sheets S_1 and S_2 carrying surface charge densities σ_1 and σ_2 respectively ($\sigma_1 > \sigma_2$) placed at a distance d apart in vacuum. Find the work done by the electric field in moving a point charge q, a distance a ($a < d$) from S_1 towards S_2 along a line making an angle $\pi/4$ with the normal to the plates.

[2]

Solution

Electric field in the space between the plates towards right

$$E = \frac{\sigma_1 - \sigma_2}{2\varepsilon_0}$$



$\therefore$ work done $W = Fs\cos\theta$

$$= qEa\cos\frac{\pi}{4}$$

$$= \frac{q(\sigma_1 - \sigma_2)a}{2\sqrt{2}\varepsilon_0}$$

4. In a Searle's experiment, the diameter of the wire as measured by a micrometer of least count 0.001 cm is 0.050 cm. The length, measured by a scale of least count 0.1 cm, is 110.0 cm. When a weight of 50 N is suspended from the wire, the extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the maximum error in the measurement of Young's modulus of the material of the wire from these data.

[2]

Solution

$$Y = \frac{FL}{Al}$$

$$Y = \frac{FL}{\left(\frac{\pi d^2}{4}\right)l}$$

$$= \frac{50 \times 110 \times 10^{-2} \times 4}{\pi \times 0.050^2 \times 10^{-4} \times 0.125 \times 10^{-2}}$$

$$= 2.24 \times 10^{11}\ \text{N/m}^2$$

$$\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta L}{L} + \frac{\Delta l}{l} = 2\left(\frac{0.001}{0.05}\right) + \frac{(0.1)}{110} + \frac{0.001}{0.125} = 0.0489$$

$\Delta Y = 0.0489$ Y

$= 10.954 \times 10^9\ \text{N/m}^2$

5. A rock is 1.5×10^9 years old. The rock contains ^{238}U, which disintegrates to form ^{206}Pb. Assume that there was no ^{206}Pb in the rock initially and it is the only stable product formed by the decay. Calculate the ratio of number of nuclei of ^{206}Pb to that of ^{238}U in the rock. Half-life of ^{238}U is 4.5×10^9 years. (given, $2^{1/3} = 1.259$)

[2]

Solution

$$^{238}\text{U} \rightarrow {}^{206}\text{Pb}$$

Initial number of nuclei	N_o	0
Final number of nuclei	N	$N_o - N$

Now $N = N_o\left(\frac{1}{2}\right)^n$ where n is the number of half-lives = $\frac{1}{3}$ in this case.

$\therefore \quad \frac{N}{N_0} = \frac{1}{2^{1/3}} \Rightarrow \frac{N_0 - N}{N} = 0.259$

6. A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity.

[2]

Solution

At terminal velocity, the viscous force acting on the sphere, $F = 6\pi\eta r v_t$ where η is the coefficient of viscosity of the liquid, r is the radius of the sphere and v_t is the terminal velocity.

Rate of production of heat at terminal velocity $\frac{dQ}{dt} = Fv_t$

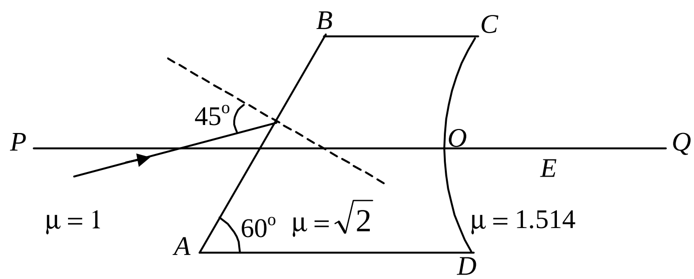
$$= 6\pi\eta r v_t \times v_t = 6\pi\eta r v_t^2 = 6\pi\eta r\left\{\frac{2}{9}\frac{r^2 g}{\eta}(\rho_s - \rho_L)\right\}^2$$

Where ρ_S and ρ_L are the densities of the sphere and liquid respectively.

$$= \left\{\frac{8\pi g^2}{27\eta}(\rho_s - \rho_L)^2\right\}.r^5$$

$$\propto r^5$$

7. Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face AB as shown in the figure. After refraction it is incident on a spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find the distance OE upto two places of decimal.



[2]

Solution

From Snell's law,

$$1 \times \sin 45^\circ = \sqrt{2}\ \sin r \quad \Rightarrow \quad r = 30^\circ$$

$\therefore$ ray becomes parallel to AD inside the block.

Now $\frac{1.514}{OE} - \frac{\sqrt{2}}{\infty} = \frac{1.514 - \sqrt{2}}{0.4}$

$\therefore$ $OE = \frac{1.514 \times 0.4}{0.1} = 6.06$ m correct upto two places of decimal.

8. The pitch of a screw gauge is 1 mm and there are 100 divisions on the circular scale. While measuring the diameter of a wire, the linear scale reads 1 mm and 47^{th} division on the circular scale coincides with the reference line. The length of the wire is 5.6 cm. Find the curved surface area (in cm^2) of the wire in appropriate number of significant figures assuming there is no zero error in the instrument. **[2]**

Solution

Least count = $\frac{1\text{ mm}}{100} = 0.01$ mm

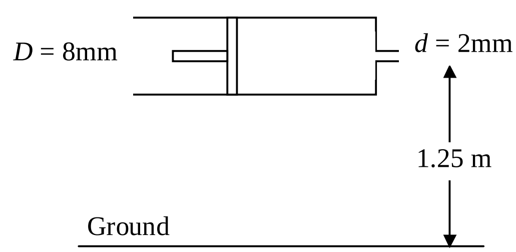
Diameter of wire = 1 mm + 47 $\times$ 0.01 mm = 1.47 mm

$\therefore$ curved surface area = $2\pi \frac{d}{2}.L = \pi dL$

$= \pi \times 1.47 \times 10^{-1} \times 5.6 = 2.5848 cm^2$

$= 2.6\ cm^2$ (in two significant figures)

9. Consider a horizontally oriented syringe containing an incompressible and non-viscous fluid located at a height of 1.25 m above the ground. The diameter of the plunger is 8 mm and the diameter of the nozzle is 2 mm. The plunger is pushed with a constant speed of 0.25 m/s. Find the horizontal range of the fluid stream on the ground. Take g = 10 m/s^2.



[2]

Solution

Equation of continuity $A_1 v_1 = A_2 v_2$

$\Rightarrow$ $\frac{\pi}{4}\left(\frac{8}{1000}\right)^2 \times 0.25 = \frac{\pi}{4}\left(\frac{2}{1000}\right)^2 v_2$ $\Rightarrow$ $v_2 = 4$ m/s

Now let x be the horizontal range then, $x = v_2 t$ and $y = \frac{1}{2} g t^2$

$\Rightarrow$ $x^2 = \frac{2 y v_2^2}{g} = \frac{2 \times 1.25 \times (4)^2}{10} = 4$ $\Rightarrow$ $x = 2$ m.

10. A proton and an alpha particle, after being accelerated through same potential difference, enter a uniform magnetic field the direction of which is perpendicular to their velocities. Find the ratio of radii of the circular paths of the two particles. **[2]**

Solution

$$r = \frac{mv}{qB} = \frac{\sqrt{2mqV}}{qB}$$

$$\therefore \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha} \cdot \frac{q_\alpha}{q_p}} = \frac{1}{\sqrt{2}}$$

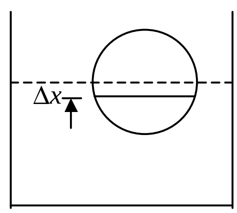
11. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[4]**

Solution

Initially, $$\rho\left(\frac{4}{3}\pi R^3\right)g = \rho_l\left(\frac{2}{3}\pi R^3\right)g$$

$$\Rightarrow \quad 2\rho = \rho_l$$

Let it be displaced a distance Δx inside the liquid.
Extra force or Restoring force,



$$F_R = -\pi R^2 \rho_l g \Delta x \text{ (area x thickness x density x g)}$$

$$\therefore \quad F_R = -k\Delta x$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

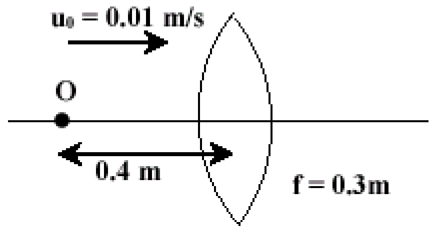
$$m = \frac{4}{3}\pi R^3 \rho$$

Putting the values of m and k we get,

$$f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$$

12. An object is approaching a thin convex lens of focal length 0.3 m with a speed of 0.01 m/s. Find the magnitudes of the rates of change of position and lateral magnification of image when the object is at a distance of 0.4 m from the lens. **[4]**

Solution



Lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ .... (1)

$\Rightarrow \quad \frac{1}{v}+\frac{1}{40}=\frac{1}{30}$

$\Rightarrow \quad v=120$ cm

Differentiating (1) with respect to time,

$$-\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$$

$\Rightarrow \quad \frac{dv}{dt}=\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$

$\Rightarrow \quad \frac{dv}{dt}=0.09$ m/s

$\therefore$ Magnitude of rate of change of position of the image = 0.09 m/s

Lateral magnification, $m = \frac{v}{u}$ .... (2)

Differentiating (2) with respect to time,

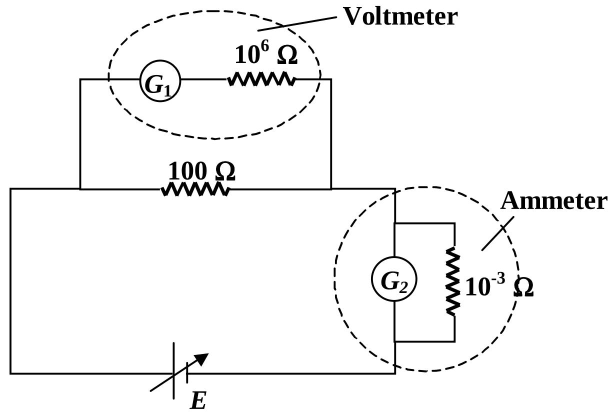
$$\frac{dm}{dt}=\frac{u.\frac{dv}{dt}-v\frac{du}{dt}}{u^2}=\frac{-0.4\times 0.09-1.2\times 0.01}{(0.4)^2}=-0.3$$

$\therefore$ Magnitude of the rate of change of lateral magnification is 0.3 s^{-1}. The –ve sign shows that it is diminishing.

13. Draw the circuit for experimental verification of Ohm's law using a source of variable D.C. voltage, a main resistance of 100 Ω, two galvanometers and two resistances of values 10^6 Ω and 10^{-3} Ω respectively. Clearly show the positions of the voltmeter and the ammeter.

[4]

Solution



14. A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between α_s and γ_l, showing all the steps.

[4]

Solution

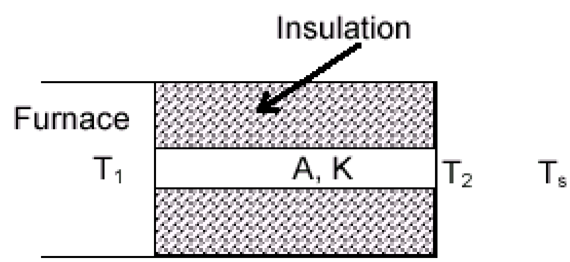
Let the base area of cube be A and depth upto which it is submerged be h and d be the density of liquid at initial temperature.

Then at final temperature density $d' = \dfrac{d}{(1+\gamma_l \Delta T)}$

and base area $A' = A(1+2\alpha_S \Delta T)$

From principle of floatation, $dAh = d'A'h \quad \Rightarrow \quad \gamma_l = 2\alpha_s$

15. One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e. It is given that $T_2 = T_S + \Delta T$ where $\Delta T << T_S$, T_S being the temperature of the surroundings. If $\Delta T \propto (T_1 - T_S)$, find the proportionality constant. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2.



[4]

Solution

Rate of heat conduction through the rod = rate of heat radiated at the right end of the rod.

$$\frac{KA(T_1 - T_2)}{L} = e\sigma A\left(T_2^4 - T_S^4\right) \cong e\sigma A 4T_S^3 (T_2 - T_S)$$

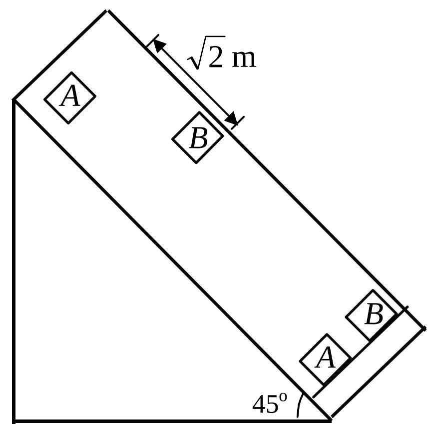
$\Rightarrow$ $$\frac{K(T_1 - T_S - \Delta T)}{L} = e\sigma 4T_S^3 \Delta T$$

$\Rightarrow$ $$\frac{K(T_1 - T_S)}{L} = \left(e\sigma 4T_S^3 + \frac{K}{L}\right)\Delta T$$

$\Rightarrow$ $$\Delta T = \frac{K(T_1 - T_S)}{\left(e\sigma 4LT_S^3 + K\right)}$$

On comparing with the given relation, proportionality constant = $\dfrac{K}{4e\sigma LT_S^3 + K}$

16. Two blocks A and B of equal masses are released from an inclined plane of inclination 45^o at $t = 0$ from rest. The coefficient of kinetic friction between the block A and the inclined plane is 0.2 while it is 0.3 for block B. Initially, the block A is $\sqrt{2}$ m behind the block B. When and where their front faces will come in a line. Take $g = 10$ m/s^2.



[4]

Solution

$$a_{rel} = a_A - a_B = g(\sin\theta - \mu_A \cos\theta) - g(\sin\theta - \mu_B \cos\theta) = \frac{1}{\sqrt{2}} \text{ m/s}^2$$

$$S_{rel} = \frac{1}{2} a_{rel} t^2$$

$\Rightarrow$ $$\sqrt{2} = \frac{1}{2}\frac{1}{\sqrt{2}}.t^2$$

$\Rightarrow$ $t = 2$ s

Distance moved by block A in this time = $\dfrac{1}{2} a_A t^2 = \dfrac{1}{2} \times g(\sin\theta - \mu_A \cos\theta)t^2$
= $8\sqrt{2}$ m

Therefore, the front faces of the blocks will come in a line after A has traveled a distance $8\sqrt{2}$ m (or B has traveled a distance $7\sqrt{2}$ m).

17. Wavelengths belonging to Balmer series lying in the range of 450 nm to 750 nm were used to eject photoelectrons from a metal surface whose work function is 2.0 eV. Find (in eV) the maximum kinetic energy of the emitted photoelectrons. Take hc = 1242 eV nm.

[4]

Solution

For Balmer series, $\frac{hc}{\lambda} = 13.6\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

Photoelectrons will have maximum KE for λ_{min}.

λ_{min} belonging to Balmer series and lying in the given range = 489 nm. (Corresponds to the transition from n = 4 to n = 2).

Therefore from Einstein's photoelectric effect equation,

$\frac{hc}{\lambda} = \phi +$ KE_{max}

KE_{max} = $\frac{hc}{\lambda_{min}} - \phi = \frac{1242}{489} - 2 = 2.54 - 2 = 0.54$ eV

18. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? Take $D/d = 10^3$. Symbols have their usual meanings. **[4]**

Solution

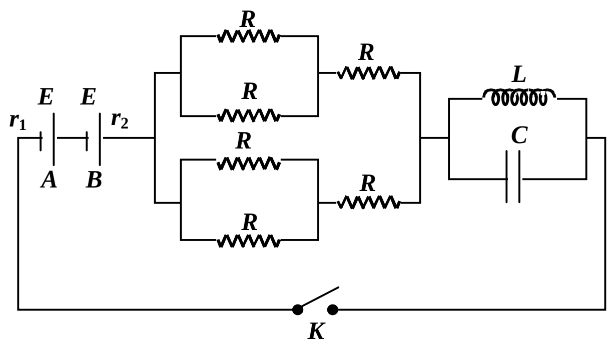
$n_1 \frac{D}{d}\lambda_1 = n_2 \frac{D}{d}\lambda_2$ (for the maxima to coincide)

$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{7}{5}$

$\Rightarrow \quad n_1 = 7, \quad n_2 = 5$

$\therefore$ Minimum distance = $n_1 \frac{D}{d}\lambda_1$ = $7 \times 10^3 \times 5 \times 10^{-7}$ = 35×10^{-4} m = 3.5 mm

19. In the circuit shown below, A and B are two cells of same emf E but different internal resistances r_1 and r_2 $(r_1 > r_2)$ respectively. Find the value of R such that the potential difference across the terminals of cell A is zero a long time after the key K is closed.



[4]

Solution

In steady state capacitor works as an open circuit and inductor works as short circuit.

$$E + E = \left(\frac{3R}{4} + r_1 + r_2\right)i$$

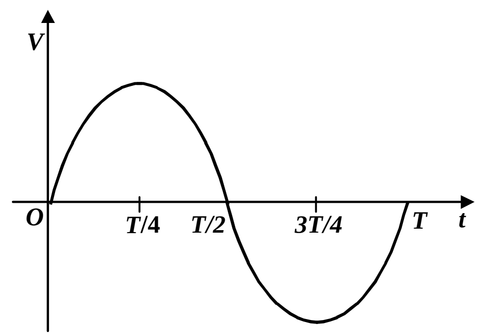
$$\Rightarrow \quad i = \frac{2E}{r_1 + r_2 + \frac{3R}{4}}$$

Now, potential difference across the terminals of cell A, $V_A = E - ir_1 = 0$

$$\Rightarrow \quad E = \frac{2E}{r_1 + r_2 + \frac{3R}{4}} r_1$$

$$\Rightarrow \quad R = \frac{4(r_1 - r_2)}{3}$$

20. In an LR series circuit, a sinusoidal voltage $V = V_o \sin \omega t$ is applied. It is given that $L = 35$ mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\frac{\omega}{2\pi} = 50$ Hz and $\pi = 22/7$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph.



[4]

Solution

We have $X_L = \omega L = 2\pi f . L = 2\,\pi \times 50 \times 35 \times 10^{-3} = 10.99 \approx 11\ \Omega$

$\therefore$ Impedance $Z = \sqrt{X_L^2 + R^2} = \sqrt{11^2 + 11^2} = 11\sqrt{2}\ \Omega$

$\therefore$ steady state current amplitude

$$i_o = \frac{V_o}{Z} = \frac{V_{rms} \times \sqrt{2}}{11\sqrt{2}} = \frac{220}{11} = 20 \text{ A}$$

Phase difference $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{11}{11}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

$\therefore$ Steady state current $i = 20 \sin \pi\left(100t - \frac{1}{4}\right)$