5. The number of solutions of the correct answers. Indicate the correct answers by A, B, C, D, E.

This section has Five Questions. Each question is provided with five alternative answers. One or more than one of them are these sections order of the questions must be maintained.

NOTE:- All answers to questions in Section-A, Section-B and Section-C must be supported by mathematical arguments. In each of these sections orders of the questions must be maintained.

SECTION-A

This section has Five Questions. Each question is provided with five alternative answers. One or more than one of them are correct answers. Indicate the correct answers by A, B, C, D, E.

(5x2=10 MARKS)

1. Let l₁, l₂ be any two parallel lines and B, C be any two points on l₁ and A₁, A₂, ..., A₅₁₀ be points on l₂. If Δ₁ denotes the area of the triangle ABC and if ∑₁⁵₁₀ Δ₁ = 2010, Then the area of Δ A₂₀₁₀BC is
   A) 1  B) ½  C) 2  D) 2010  E) 1005

2. Let {aₙ} be a sequence of integers such that a₁ = 1, aₙ₊₁ = aₙ + mn for all positive integers m and n. Then a₁₂ is
   A) 6  B) 70  C) 78  D) 76  E) 72

3. In a triangle ABC, a, b, c denote the lengths of the sides BC, CA, AB. If D is the midpoint of the side BC and AD is perpendicular to AC, then
   A) 3b² = a² – c²  B) 3a² = b² – 3c²  C) b² = a² – c²  D) a² + b² = 5c²  E) none of these

4. If k is an integer then which of the following is true?
   A) An integer of the form 4k+1 can always be put in the form 2k-1
   B) An integer of the form 4k+3 can always be put in the form 2k+1
   C) An integer of the form 2k-1 can always be put in the form 4k+1
   D) An integer of the form 2k+1 can always be put in the form 4k+3
   E) An integer of the form 2k+1 can always be put in the form 4k+3

5. The number of elements in {(a, b, c) / a=b, (a-c)²=0, a, b, c are real numbers} is
   A) 0  B) 1  C) 6  D) 3  E) infinitely many

SECTION-B

This section has Five Questions. In each question a blank is left. Fill in the blank.

(5x2=10 MARKS)

1. The no. of solutions of the equation x₁(x₁+y₁)=2010, where x and y denote positive prime numbers, is

2. The number of elements in the set {n² / n²-8n²+20n-13 is a prime number} is

3. The solution set of the equation √x³ – 4x + 4 = (x-2) = 0 is

4. Given any two diameters of a circle the convex quadrilateral formed by joining the extremities of the diameters is always a rectangle. True/False

5. If P = 3²⁰¹⁰ + 3²⁰¹⁰, Q = 3²⁰¹⁰ – 3²⁰¹⁰ then P² – Q² =

SECTION-C

(5x2=10 MARKS)

1. Solve the equation log₂⁰¹⁰ (2009x) = log₂⁰₀⁵ (2010x).

2. In a quadrilateral ABCD, AB = 3, BC = 4, CD = 5, ∠ABC = ∠BCD = 120°. Find the area of the quadrilateral.

3. I was trying to solve 4/x = 5. While writing the question I mistakenly wrote a digit other than 5 and solved the inequality and got 2<x<4. What digit did I write possibly?

4. In a right angled triangle what is the relation between the square of the altitude on to the hypotenuse and the product of the segments of the hypotenuse?

5. Is it possible to find two functions f and g such that the domain of f is not finite, the domain of g is finite, gof is defined? Justify your answer.

SECTION-D

(5x4=20 MARKS)

1. If the last digits (unit places) of the products 1.2, 2.3, 3.4, ..., n(n+1) are added, the result is 2010. How many products are used?

2. Show that four divides any perfect square or leaves a remainder 1. Also show that nine divides cube of any integer or leaves 1 or 8 as remainder.

3. Let AB be a line segment of length 26. Let C and D be located on the line segment AB such that AC = 1 and AD = 8. Let E and F be the points on one of the semi circles with diameter AB for which EC and FD are perpendicular to AB. Find the length of the line segment EF.

4. In each of the following cases give an example of a system of two linear equations in two variables x and y.
   i) A system having exactly one solution
   ii) A system having no solution
   iii) A system having infinitely many solutions

5. Using Mathematical Induction Prove that 3ⁿ + 7 is divisible by 8, ∀ n ∈ N.