**MATHEMATICS**

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| **Time: Two Hours**  **(8.30 AM – 10.30 AM)** | **Max.Marks:60** |

**NOTE:-**

1. *Attempt all questions.*
2. *Answer to each question should begin on a fresh page, and rough work must be enclosed with answer book.*
3. *There is no negative marking.*
4. *Answer all parts of a question at one place.*
5. *Use of calculators, slide rule, graph paper and logarithmic, trigonometric and statistical tables is not permitted.*

**Note:-**All answers to questions in **Section-A, Section-B, Section-C**must be supported by mathematical arguments. In each of these sections order of the parts must be maintained.

**SECTION-A**

1. **This section has Five questions. Each question is provided with five alternative answers. Only one of them is the correct answer. Indicate the correct answer by A,B,C,D,E.**(5 2=10 MARKS)

1. Let m be the l.c.m. of 32002-1 and 32002+1. Then the last digit of m is

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| A) 0 | B) 4 | C) 5 | D) 8 | E) none |

Hint: Read off g.c.d. and hence determine the l.c.m.

2. In every cyclic quadrilateral ABCD with AB parallel to CD

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| A) AD = BC, AC http://surendranath.tripod.com/Sat/Sat02/Mat/Image1.gifBD | B) ADhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image1.gifBC, AC http://surendranath.tripod.com/Sat/Sat02/Mat/Image1.gifBD |
| C) AD http://surendranath.tripod.com/Sat/Sat02/Mat/Image1.gif BC, AC = BD | D) AD = BC, AC = BD |
| E) none of these |  |

3. f,g,h:Rhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image2.gifR are defined by f(x) = x-1, g(x) = http://surendranath.tripod.com/Sat/Sat02/Mat/Image3.gif if xhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image1.gif-1 h(x) = http://surendranath.tripod.com/Sat/Sat02/Mat/Image4.gif if xhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image1.gif-1, xhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image1.gif-2 -2 if x = -1 -2 if x = -1 -3 if x = -2 Then f(x) + g(x) – 2.h(x) is

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| A) not a polynomial | B) a polynomial whose degree is undefined |
| C) a polynomial of degree 0 | D) a polynomial of degree 1 |
| E) none of these |  |

4. A, B are two distinct points on a circle with center C1 and radius r1. http://surendranath.tripod.com/Sat/Sat02/Mat/Image5.gif also is a chord of a different circle with center C2 and radius r2. S denotes the statement: The arc AB divides the first circle into two parts of equal area.

A) S happens if C1C2 > r2

B) If S happens then C1C2 > r2

C) If S happens then it is possible that C1C2 = r2

D) If S happens then it is necessary that C1C2 < r2

E) none of these

5. Triangle T1 has vertices at (-3,0), (2,0) and (0,4). Triangle T2 has vertices at (-3,3), (3,5) and (1,-2). Plot all points lhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image6.gifm where the line segment l is a side of T1 and the line segment m is a side of T2, whenever lhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image6.gifm is a unique point. These points of intersection determine a convex polygon, which is a

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| A) triangle | B) quadrilateral | C) pentagon | D) hexagon | E) septagon |

**SECTION-B**

**II. This section has Five questions. In each question a blank is left. Fill in the blank.**(5 2=10 MARKS)

1. The number of integral solutions of the equation x4-y4 = 2002 is \_\_\_\_\_

2. A,B,C,D are distinct and concyclic. http://surendranath.tripod.com/Sat/Sat02/Mat/Image7.gif is perpendicular to http://surendranath.tripod.com/Sat/Sat02/Mat/Image8.gif, http://surendranath.tripod.com/Sat/Sat02/Mat/Image9.gifis perpendicular to http://surendranath.tripod.com/Sat/Sat02/Mat/Image10.gif. http://surendranath.tripod.com/Sat/Sat02/Mat/Image11.gif intersects http://surendranath.tripod.com/Sat/Sat02/Mat/Image12.gifin N. The nonobtuse angle enclosed between http://surendranath.tripod.com/Sat/Sat02/Mat/Image12.gif and http://surendranath.tripod.com/Sat/Sat02/Mat/Image13.gif is \_\_\_\_\_ in degrees

3. Let m be the 2002-digit number each digit of which is 6. The remainder obtained when m is divided by 2002 is \_\_\_\_\_\_

4. If x is a real number Int.x denotes the greatest integer less than or equal to x.

Int.http://surendranath.tripod.com/Sat/Sat02/Mat/Image14.gif = \_\_\_\_\_\_\_\_\_\_

5. O is the origin and Pi is a point on the curve x2 + y2 = i2 for any natural number i.

http://surendranath.tripod.com/Sat/Sat02/Mat/Image15.gif = OPj, where j = \_\_\_\_\_\_\_\_\_

**SECTION-C**

**III. This section has Five questions. Each question has a short answer. Elegant solutions will be rewarded.**(5 2=10 MARKS)

1. If m is the right most nonzero digit of (n!)4 where n is a positive integer greater than 1, determine the possible values of m.

2. Sketch the graph of the function f(x) = http://surendranath.tripod.com/Sat/Sat02/Mat/Image16.gif , http://surendranath.tripod.com/Sat/Sat02/Mat/Image17.gif.

3. http://surendranath.tripod.com/Sat/Sat02/Mat/Image18.gifABC is right angled at A. AB=60, AC=80, BC=100 units. D is a point between B and C such that the triangles ADB and ADC have equal perimeters. Determine the length of AD.

4. Do there exist positive integers p, q, r such that http://surendranath.tripod.com/Sat/Sat02/Mat/Image19.gif?

5. k is a constant, (xi,0) is the mid point of (i-k, 0) and (i+k, 0) for i=1,2,…,2002 then find http://surendranath.tripod.com/Sat/Sat02/Mat/Image20.gif.

**SECTION-D**

**IV. This section has Five questions. These questions are long answer questions. Elegant solutions will be rewarded.**(5 6=30 MARKS)

1. If m and n are natural numbers such that

http://surendranath.tripod.com/Sat/Sat02/Mat/Image21.gif.

Prove that 2002 divides m.

2. http://surendranath.tripod.com/Sat/Sat02/Mat/Image18.gifABC is right angled at A. Prove that the three mid points of the sides and the foot of the altitude through A are concyclic.

3. Determine all possible finite series in GP with first term 1, common ratio an integer greater than 1 and sum 2002.

4. A convex pentagon ABCDE has the property that the area of each of the five triangles ABC, BCD, CDE, DEA, and EAB is unity. Prove that the area of the pentagon is http://surendranath.tripod.com/Sat/Sat02/Mat/Image22.gif .

5. If a,b,c, are real numbers such that the sum of any two of them is greater than the third andhttp://surendranath.tripod.com/Sat/Sat02/Mat/Image23.gif=3abc, then prove that a=b=c.

6. Explain how you will construct a triangle of area equal to the area of a given convex quadrilateral. Use only rough sketches and give proof.