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Serial No. 1511 **C–JTT–J–TD**

STATISTICS–IV

Time Allowed : Three Hours *Maximum Marks : 200*

INSTRUCTIONS

*Candidates should attempt **FIVE** questions in all including Question Nos. 1 and 5 which are compulsory and attempt remaining **THREE** questions by choosing at least **ONE** each from Sections–A and B.*

The number of marks carried by each question is indicated at the end of the question.

*Answers must be written in **ENGLISH**.*

Symbols and abbreviations are as usual.

If any data is required to be assumed for answering a question, it may be suitably assumed, indicating this clearly.

SECTION—A

1. Attempt any **FIVE** parts :—

   (a) Show that in an irreducible Markov chain, all the states are of same type. 8

   (b) The inter-arrival time of customers in M | M | 1 queueing system follows negative exponential distribution with parameter the arrival rate of the system. Prove this. 8

   (c) Find the differential equation of pure death process. 8

   (d) Explain the terms :—

      (i) Convex set

      (ii) Optimum basic feasible solution

      (iii) Basic feasible solution

      (iv) Degeneracy. 8

   (e) Describe the basic EOQ inventory model. 8

   (f) Explain the geometric interpretation of the Simplex procedure. 8

2. (a) Show that a Markov chain remains Markov if the time is reversed. 10

   (b) Define :

      (i) Random walk with absorbing barriers

      (ii) Random walk with reflecting barriers

(iii) Random walk with elastic barriers

(iv) Correlated random walk. 10

(c) A barber opens up for business at $t = 0$. Customers arrive at random in a Poisson fashion; that is, the pdf of interarrival time is $f(t) = \lambda e^{-\lambda t}$. Each haircut takes X seconds (where 'X' is a random variable).

Find the probability that the second arriving customer will not have to wait and also find 'W', the average value of his waiting time for the following two cases :

(i) X = constant = c

(ii) X is exponentially distributed with

$$\text{pdf} : g(x) = \mu \exp(-\mu x).$$ 10

(d) Consider the discrete-state, discrete-time Markov Chain whose transition probability matrix is given by

$$\mathbf{P} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

Find :

(i) The stationary state probability vector $\boldsymbol{\pi}$,

(ii) $[\mathbf{I} - z\mathbf{P}]^{-1}$ and

(iii) $\mathbf{P}^n$. 10

 (Contd.)

3. (a) If the objective function of a linear programming problem possesses a finite maximum, show that at least one optimal solution is a basic feasible solution. 10

(b) Describe the economic interpretation of the primal-dual problems. 10

(c) If there exists a stationary distribution, show that it is necessarily unique. 10

(d) Using duality theory, solve the following L.P. problem :—

Minimize $Z = 2x_1 + 2x_2$

subject to

$$2x_1 + 4x_2 \geq 1$$
$$x_1 + 2x_2 \geq 1$$
$$2x_1 + x_2 \geq 1$$
$$x_1, x_2 \geq 0.$$

10

4. (a) Let $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ be two independent homogeneous Poisson processes with means $\lambda_1 t$ and $\lambda_2 t$ respectively. Define the process $\{N(t), t \geq 0\}$, where $N(t) = N_1(t) - N_2(t)$.

Find $P[N(t) = n]$ for any integer n. 10

(b) Let $\{X_n, n = 0, 1, 2, ...\}$ denote a branching process

such that $X_{n+1} = \sum_{i=1}^{n} Z_i$,

where Z_{is} are iid with distribution $\{p_k\}$. Define $E(Z_1) = m$ and $Var(Z_1) = \sigma^2$.

(Contd.)

Then show that :

(i) $E(X_n) = m^n$

(ii) $Var(X_n) = \sigma^2 m^{n-1} \frac{(m^n - 1)}{m - 1}$, if $m \neq 1$

$= n\sigma^2$, if $m = 1$. 10

(c) Describe two methods for generating random variables for given continuous probability distribution. 10

(d) Introduce the concept of individual and group replacement policies. Derive the optimal group replacement policy under the usual setup. 10

SECTION—B

5. Attempt any **FIVE** parts :—

(a) State briefly how you can use the abridged life table to prepare a complete life table. 8

(b) Explain 'push-pull-obstacle' theory of migration. 8

(c) Show that in a stationary population, the birth rate is the reciprocal of the expectation of life at birth. 8

(d) Show that the age distribution of two stable populations would be identical if they are such that their mortality at all ages differs by a constant factor. 8

(e) Explain the following :—

   (i) Data hierarchy

   (ii) Data integrity

   (iii) Data logging. 8

(f) Draw a flowchart to arrange 'n' numbers in the ascending order. 8

6. (a) Describe the various methods for obtaining intercensal and post-censal estimates of population total. 10

(b) What are the different measures of mortality ? Discuss the merits and demerits of each. 10

(c) Give a short account of the main findings of the population census of India, 2001. 10

(d) Complete the following life table :—

Age (years)	l_x	d_x	q_x	L_x	T_x	e_x^o
65	4412	—	—	—	—	—
66	3724	—	—	—	—	—
67	3201	642	—	—	26567	—

10

7. (a) Describe the indirect measures of net internal migration. 10

(b) Distinguish between sequential files and direct files. 10

(c) What do you understand by a Spreadsheet Package ? Describe the features of a popular Spreadsheet Software Package. 10

(d) Explain the following :—

(i) Binary and octal number systems

(ii) Database administrator

(iii) Database language

(iv) Data logging. 10

8. (a) Derive the logistic model of population growth. Find the final size of the total population with this model. 10

(b) Derive the characteristic equation of stable population stating the underlying assumptions. 10

(c) Explain the following :—

(i) Client-server architecture

(ii) Multiprogramming

(iii) Development of Personal Computer. 10

(d) Explain classification of Algorithms with examples. 10