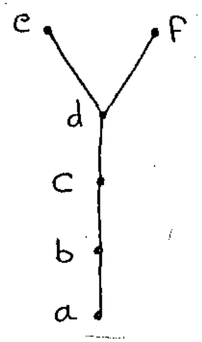
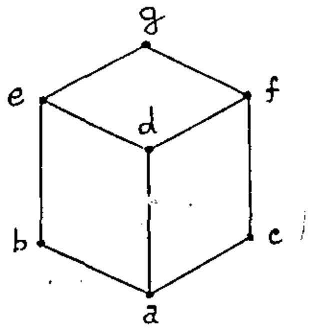
IX. (b) Find all maximal and minimal elements of the poset from the Hasse diagram.



OR

X. (a) Define Lattice and determine whether the Hasse diagram given below represents a lattice.



(b) Let L be a lattice. Then for every a and b in L prove that $a \vee b = b$ if and only if $a \leq b$.

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B.Tech. Degree III Semester (Lateral Entry) Examination, April 2003

IT/CS 303 DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours

Maximum Marks: 100

(All questions carry ***EQUAL*** marks)

I. Define "characteristic function" and show that if A and B are subsets of a universal set U.

(i) $F_{A\cap B}(x) = F_A(x)\cdot F_B(x)$ for all x.

(ii) $F_{A\cap B}(x) = F_A(x) + F_B(x) - F_A(x)\, F_B(x)$ for all x.

(iii) Prove that $2^n > n^3$ for $n \geq 10$.

OR

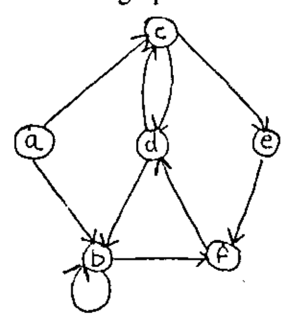
II. (a) Using equivalence formulas prove the following is a tautology:

$$\left[(P\vee Q)\wedge \sim(\sim P\wedge(\sim Q\vee \sim R))\right]\vee(\sim P\wedge \sim Q)\vee(\sim P\wedge \sim Q)$$

(b) How many different seven-person committees can be formed each containing 3 women from an available set of 20 women and 4 men from an available set of 30 men?

(c) A fair coin is tossed five times. What is the probability of obtaining three heads and two tails?

III. (a) Let R be a relation whose digraph is shown below:



(i) Find M_R^2.

(ii) Find R^∞.

(Turn over)

III. (b) Show that if R_1 and R_2 are equivalence relations on A then $R_1 \cap R_2$ is an equivalence relation.

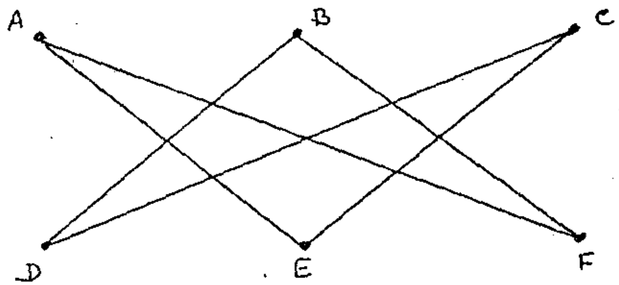
OR

IV. (a) Let $A = \{a, b, c\}$. Determine whether the relation R whose matrix M_R is given is an equivalence relation.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) Let $F : A \rightarrow B$ and $g : B \rightarrow C$ be invertible functions. Prove that $g \circ F$ is invertible and $(g \circ F)^{-1} = F^{-1} \circ g^{-1}$.

V. (a) Use Fleury's algorithm to produce an Euler circuit for the graph given below:



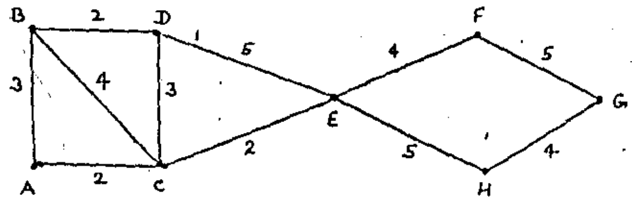
(b) Prove that if a graph G has more than two vertices of odd degree, then there can be no Euler path in G.

OR

VI. (a) Define the following and illustrate them through examples: Connected graph, complete graph and regular graph.

Contd............3.

(b) Using prim's algorithm find a minimal spanning tree for the connected graph shown below: (Begin with 'E' as the initial vertex).



VII. (a) Determine whether the set Z^* is a semigroup, where '$*$' is defined as ordinary multiplication.

(b) Let G be a group of non-zero real numbers for multiplication and $\bar{G} = \{1, -1\}$ for multiplication.

Define $\phi : G \rightarrow \bar{G}$ by $\phi(x) = 1$ if x is positive

$\phi(x) = -1$ if x is negative.

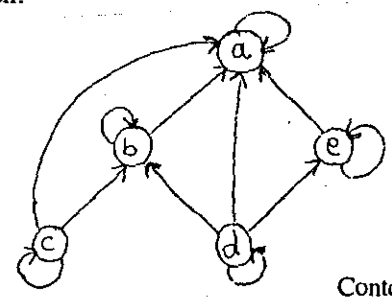
Show that 'ϕ' is a homomorphism.

OR

VIII. (a) Show that the set N of natural numbers is a semigroup under the operation $x * y = \max(x, y)$. Is it monoid?

(b) Show that the set Q_1 of all rational numbers other than 1, with the operation denoted by $a * b = a + b - ab$ consists an abelian group.

IX. (a) Determine the Hasse diagram of the partial order having the given digraph:



Contd.................4.