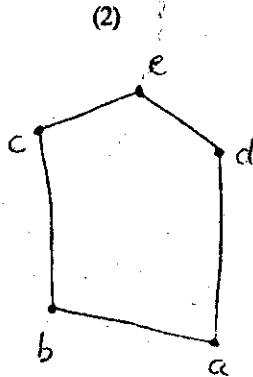X   a)   Define a lattice.

b)   Which of the following diagrams represent a lattice; justify your answer.

(1)   (2)

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B.Tech. Degree III Semester Examination, January 2002

IT/CS 303 DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours   *Max. Marks: 100*

(All questions carry equal marks)

I   Prove the following:-

(i)   Let A = $\{a,b,c,d,e\}$; B=$\{c,e,f,g,h,k,m\}$,

then $|A \cup B| = |A| + |B| - |A \cap B|$

(ii) $(p \rightarrow q) \equiv ((\sim p) \vee q)$

(iii) $1 + 2 + 3 + \dots + n < \dfrac{(2n+1)^2}{8}$

OR

II   a)   Is the following equivalence a true statement?
             $3 > 2$ if and only if $0 < 3 - 2$

b)   A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of the selected balls will be red and two will be green?

III   Let A = {a,b,c,d,e} and R = {(a,a), (a,b), (b,c), (c,e), (c,d),(d,e)}, compute (1) R^2 (2) R^{∞}
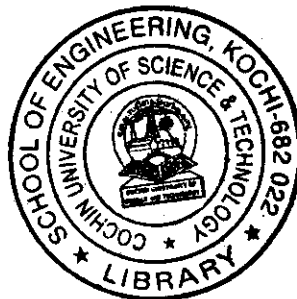
OR

IV   a)   Let A = {a,b,c,d} and R be the relation on A that has the

matrix $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Construct the digraph of R and list in-degrees and out-degrees of all vertices.

(P.T.O)

IV b) Let A = B = Z and C be the set of all even integers.
Let f : A → B and g : B → C be defined by
$$f(a) = a + 1$$
$$g(b) = 2b$$
compute g o f.

V a) Define a Hamiltonian circuit in G. Draw a graph which has Hamiltonian path but not Hamiltonian circuit.

b) Which of the following graphs have an Euler circuit, an Euler path but not an Euler circuit or neither.
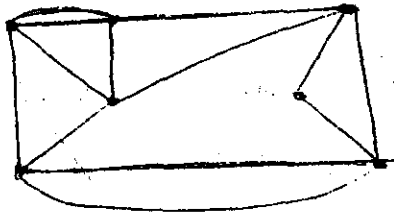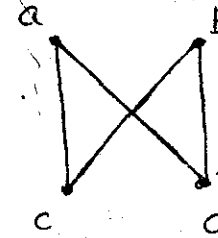

fig.(1)   (fig.2)

fig.(3)

OR
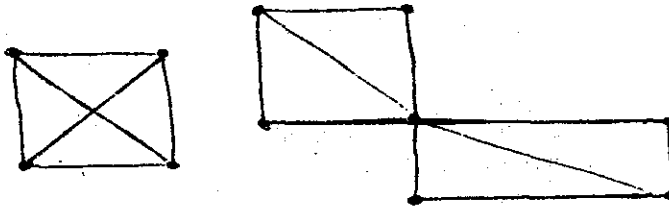
VI Using Prim's algorithm find a minimal spanning tree for the graph shown below. Choose a as the initial vertex.

VII Let A = {a, b}. Which of the following tables define a semi group on A? Which define a monoid on A?

(i)*	a	b	(ii)*	a	b	(iii)*	a	b	(iv)*	a	b
a	a	b	a	a	a	a	b	b	a	a	b
b	b	a	b	b	b	b	a	a	b	b	b

OR

VIII Let G be the set of all non-zero real numbers and

let $a*b = \dfrac{ab}{2}$. Then show that (G, *) is an Abelian group.

IX Let A = {a,b,c,d,e,f}. Draw Hasse diagram of a poset (A, ≤) and also of the dual poset (A, ≥).

OR