PART – I: CHEMISTRY

PAPER - II

SECTION – I

Straight Objective Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

(*) marked ***questions are from 11th syllabus.***

1. For a first order reaction A → P, the temperature (T) dependent rate constant (k) was found to follow the equation $\log k = -(2000)\frac{1}{T} + 6.0$. The pre-exponential factor A and the activation energy E_a, respectively, are

(A) $1.0 \times 10^6 s^{-1}$ and 9.2 kJ mol^{-1}
(B) 6.0 s^{-1} and 16.6 kJ mol^{-1}
(C) $1.0 \times 10^6 s^{-1}$ and 16.6 kJ mol^{-1}
(D) $1.0 \times 10^6 s^{-1}$ and 38.3 kJ mol^{-1}

Key. **(D)**

Sol. $k = Ae^{-Ea/RT}$

$\log k = \log A - \frac{E_a}{2.303RT}$

Log A = 6, A = $10^6 s^{-1}$

$-\frac{E_a}{2.303 \times 8.3 \times T} = \frac{2000}{T}$ –

$E_a = 2000 \times 2.303 \times 8.3$ J

= 38.3 kJ

2. The spin only magnetic moment value (in Bohr magneton units) of $Cr(CO)_6$ is

(A) 0
(B) 2.84
(C) 4.90
(D) 5.92

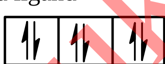
Key. **(A)**

Sol. $Cr(CO)_6$

Cr(zero)

Atomic configuration : $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1\ 3d^5$

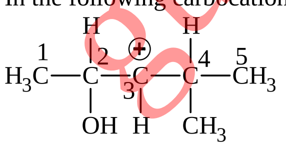
CO is a strong field ligand

∴ Configuration  t_{2g}

No. of unpaired electron = 0

∴ magnetic moment = 0

*3. In the following carbocation, H/CH_3 that is most likely to migrate to the positively charged carbon is



(A) CH_3 at C-4
(B) H at C-4
(C) CH_3 at C-2
(D) H at C-2

Key. **(D)**

Sol. Hydride shift from C-2 will yield resonance stabilized 2°-carbocation giving thereby ketonic product after deprotonation.

*4. The correct stability order of the following resonance structures is

$H_2C=\overset{+}{N}=\overset{-}{N}$ (I) $H_2\overset{+}{C}-N=\overset{-}{N}$ (II) $H_2\overset{-}{C}-\overset{+}{N}\equiv N$ (III) $H_2\overset{-}{C}-N=\overset{+}{N}$ (IV)

(A) (I) > (II) > (IV) > (III)
(B) (I) > (III) > (II) > (IV)
(C) (II) > (I) > (III) > (IV)
(D) (III) > (I) > (IV) > (II)

Key. **(B)**

Sol. In I and III all the atoms fulfil the octet requirement.
Between II and IV, structure II has negative charge on nitrogen atom. Whereas in IV –ve charge occurs at carbon which is less electronegative.

SECTION – II

Multiple Correct Answer Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

5. For the reduction of NO_3^- ioin in an aqueous solution E is +0.96 V. Values of E° for some metal ions are given below

V^{2+} (aq) + $2e^- \rightarrow V$ $E^0 = -1.19V$

Fe^{3+} (aq) + $3e^- \rightarrow Fe$ $E^0 = -0.04V$

Au^{3+} (aq) + $3e^- \rightarrow Au$ $E^0 = +1.40$ V

Hg^{2+} (aq) + $2e^- \rightarrow Hg$ $E^0 = +0.86V$

The pair(s) of metals that is(are) oxidized by NO_3^- in aqueous solution is(are)

(A) V and Hg (B) Hg and Fe

(C) Fe and Au (D) Fe and V

Key. **(A, B, D)**

Sol. NO_3^- ion will oxidise all those metal ions whose $E^{o}_{reduction}$ is less than 0.96V

*6. Among the following, the state function(s) is(are)

(A) Internal energy (B) Irreversible expansion work

(c) Reversible expansion work (d) Molar enthalpy

Key. **(A, D)**

Sol. ΔE and ΔH are path independent and are definite quantities in a given change of states. Hence, E and H are state function.

***7.** In the reaction

$2X + B_2H_6 \rightarrow [BH_2(X)_2]^+ [BH_4]^-$

The amine(s) X is(are)

(A) NH_3 (B) CH_3NH_2

(C) $(CH_3)_2NH$ (D) $(CH_3)_3N$

Key. **(A, B, C)**

Sol. 3°-Amine form some different kind of complex with diborance

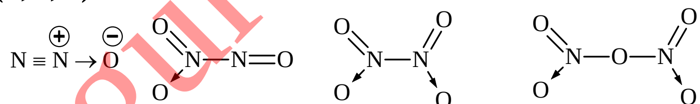
8. The nitrogen oxide(s) that contain(s) N-N bond(s) is(are)

(A) N_2O (B) N_2O_3

(C) N_2O_4 (D) N_2O_5

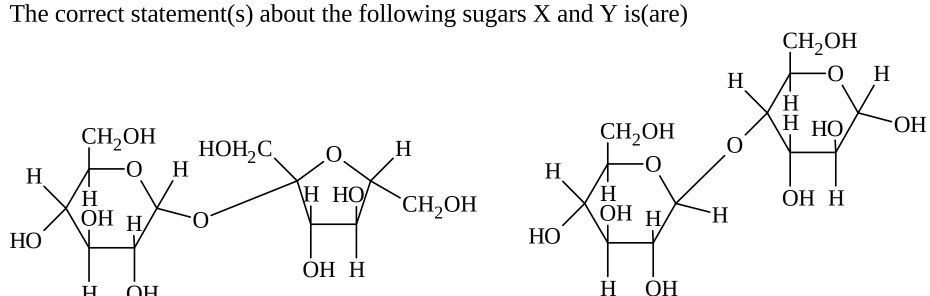
Key. **(A, B, C)**

Sol.



(A) (B) (C) (D)

9. The correct statement(s) about the following sugars X and Y is(are)



X Y

(A) X is a reducing sugar and Y is a non-reducing sugar
(B) X is a non-reducing sugar and Y is a reducing sugar
(C) The glucosidic linkages in X and Y are α and β, respectively
(D) The glucosidic linkages in X and Y are β and α, respectively

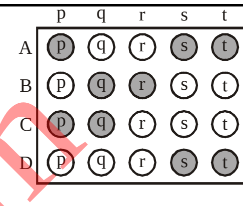
Key. **(B, C)**

Sol. In "X" the glycosidic linkage is inbetween two anomeric C-atom while in Y it is only with one anomeric carbon, the other one is free. So, "X" will be non-reducing while "Y" will be reducing. Again the glycosidic linkage in X is in between α-glucose and α-fructose, In Y, one of the glucose unit is α.
Hence (B) and (C)

SECTION – III

Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*
If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



10. Match each of the reactions given in Column I with the corresponding product(s) given in Column II.

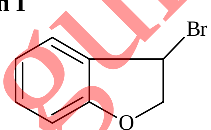
Column I		**Column II**	
(A)	Cu + dil HNO_3	(p)	NO
(B)	Cu + conc HNO_3	(q)	NO_2
(C)	Zn + dil HNO_3	(r)	N_2O
(D)	Zn + conc HNO_3	(s)	$Cu(NO_3)_2$
		(t)	$Zn(NO_3)_2$

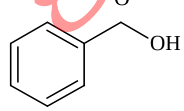
Key. **(A – p, s), (B – q, s), (C – r, t), (D – q, t)**

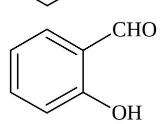
Sol.
$$3Cu + \underset{(dil)}{8HNO_3} \longrightarrow 3Cu(NO_3)_2 + 4H_2O + 2NO\uparrow$$
$$Cu + \underset{(conc)}{4HNO_3} \longrightarrow Cu(NO_3)_2 + 2H_2O + 2NO_2\uparrow$$
$$4Zn + \underset{(dil.)}{10HNO_3} \rightarrow 4Zn(NO_3)_2 + 5H_2O + N_2O\uparrow$$
$$Zn + \underset{(conc)}{HNO_3} \rightarrow Zn(NO_3)_2 + H_2O + NO_2\uparrow$$

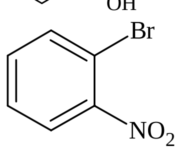
11. Match each of the compounds given in Column I with the reaction(s), that they can undergo, given in Column II.

Column I

(A) 

(B) 

(C) 

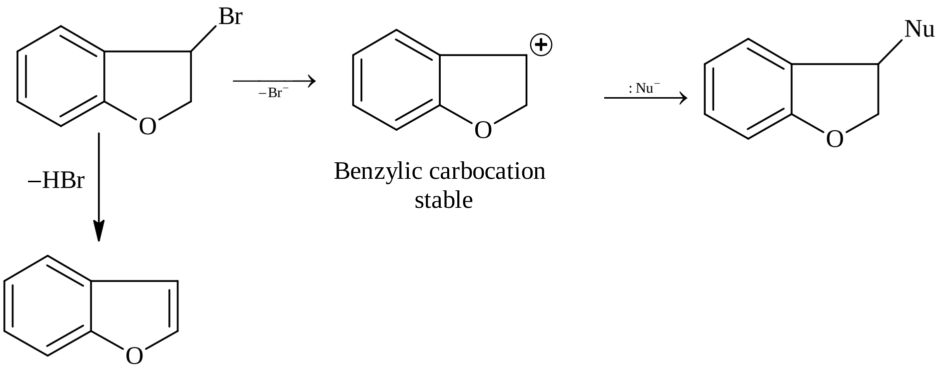
(D) 

Column II

(p) Nucleophilic substitution
(q) Elimination
(r) Nucleophilic addition
(s) Esterification with acetic anhydride
(t) Dehydrogenation

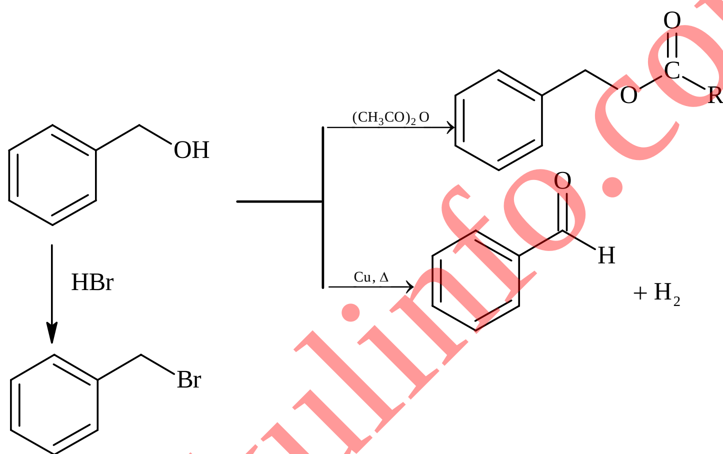
Key. **(A – p, q), (B – p, s, t), (C – r, s), (D – p)**

(A)

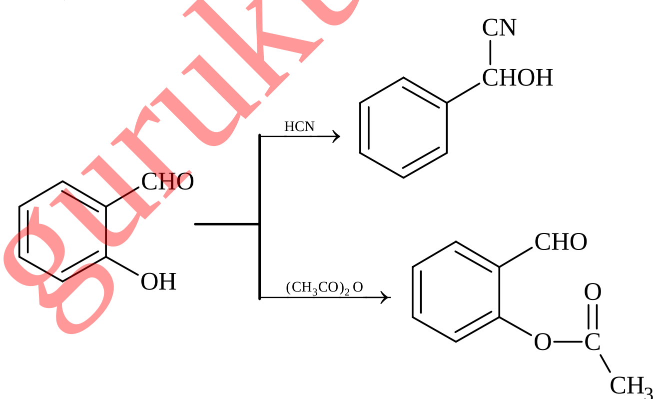


The alkenic d bond is conjugated with the aromatic nucleus

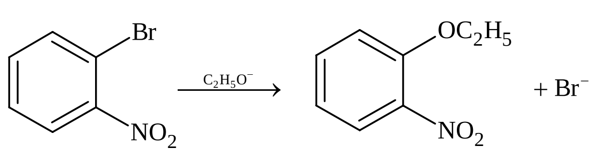
(B)



(C)



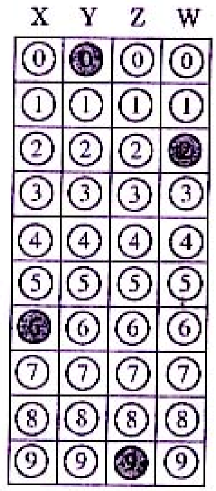
(D)



SECTION – IV

Integer Answer Type

This section contains 8 questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



*12. In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K^{-1}, the numerical value for the enthalpy of combustion of the gas in kJ mol^{-1} is

Key. 9 kJ mol^{-1}

Sol. Rise in temperature (293.45 – 298)
= 0.45 K

$\therefore$ Heat evolved = 0.45 × 2.5 = 1.125 kJ

$\therefore$ No. of moles $\frac{3.5}{28} = \frac{1}{8}$ mol

$\therefore$ Enthalpy of combustion
= 8 × 1.125
= 9 kJ/moles

*13. At 400K, the root mean square (rms) speed of a gas X (molecular weight = 40) is equal to the most probable speed of gas Y at 60 K. The molecular weight of the gas Y is

Key. 4 $gmol^{-1}$

Sol. $U_{rms} = \sqrt{\frac{3RT}{M}}$

$U_{mp} = \sqrt{\frac{2RT}{M}}$

$\therefore$ From questions

$$\sqrt{\frac{3R\times 400}{40}} = \sqrt{\frac{2R\times 60}{M}}$$

M = 4

*14. The dissociation constant of a substituted benzoic acid at 25°C is 1.0×10^{-4}. The pH of a 0.01 M solution of its sodium salt is

Key. 8

Sol. $pH = 7 + \frac{1}{2}pK_a \quad \frac{1}{2}\log C$

$= 7 + \frac{1}{2}\ 4 \times \frac{1}{2}\log 0.01$

$= 8$

*15. The total number of α and β particles emitted in the nuclear reaction ${}^{238}_{92}U \rightarrow {}^{214}_{82}Pb$ is

Key. 8

Sol. ${}_{92}U^{238} \longrightarrow {}_{82}Pb^{214}$

No. of α particle = $\frac{238-214}{4}$

$= \frac{24}{4} = 6 \quad \alpha$

No. of β particle = 2β

Total particle = 6 + 2 = 8

16. The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is

Key. 6

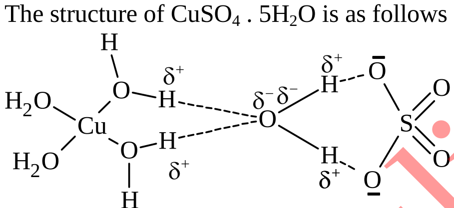
Sol. $MnO_2 + 2KOH + \frac{1}{2}O_2 \longrightarrow K_2MnO_4 + H_2O$

Oxidation state of Mn is +6

17. The number of water molecule(s) directly bonded to the metal centre in $CuSO_4 . 5H_2O$ is

Key. 4

Sol. The structure of $CuSO_4 . 5H_2O$ is as follows



That is only four water molecules are coordinated to central Cu^{2+} ion. One H_2O molecule exists H-bonded. Hence answer is 4.

*18. The coordination number of Al in the crystalline state of $AlCl_3$ is

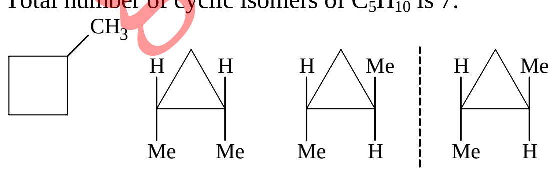
Key. 6

Sol. At low temperature $AlCl_3$ exists a closed packed lattice of Cl^- ions having Al^{3+} ion in octahedral void. Hence C.N. is six.

*19. The total number of cyclic structural as well as stereo isomers possible for a compound with the molecular formula C_5H_{10} is

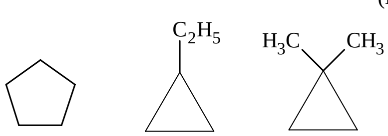
Key. 7

Sol. Total number of cyclic isomers of C_5H_{10} is 7.



cis and meso (only one) trans- (+) trans-(-)

(Non-superimposable mirror image)



PART – II: MATHEMATICS

SECTION – I

Straight Objective Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

20.

***20.** If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Key (C)

Sol.: $T_n = S_n - S_{n-1} = c[n^2 - (n-1)^2] = c\,(2n-1)$

$\Rightarrow$ Required sum $= c^2\sum_{r=1}^{n}(4r^2-4r+1) = c^2\left[\frac{4n(n+1)(2n+1)}{6}-\frac{4n(n\quad 1)}{2}+n\right]$ +

$= \frac{n(4n^2-1)c^2}{3}$.

21. A line with positive direction cosines passes through the point P(2, -1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals

(A) 1 (B) $\sqrt{2}$

(C) $\sqrt{3}$ (D) 2

Key (C)

Sol.: Let the line make the angle α with the axes, then we have

$3\cos^2\alpha = 1$ [$\because$ sum of the square's of DC's = 1]

$\cos\alpha = \frac{1}{\sqrt{3}}$ [$\because$ DC's are positive, given]

$\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z\quad 2}{1\sqrt{3}} = r$ {where $r = \overline{PQ}$}

$x = \frac{r}{\sqrt{3}}+2,\ y = \frac{r}{\sqrt{3}}-1,\ z = \frac{r}{\sqrt{3}}+2$

$\frac{2r}{\sqrt{3}}+4+\frac{r}{\sqrt{3}}\quad 1\ \frac{r}{\sqrt{3}}+2\quad 9\ +\qquad =$

$\frac{4r}{\sqrt{3}} = 4 \Rightarrow r = \sqrt{3}$

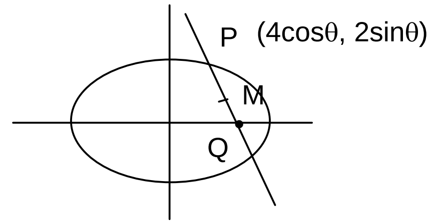
***22.** The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

(A) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2},\pm\frac{\sqrt{19}}{4}\right)$

(C) $\left(\pm 2\sqrt{3},\pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3},\pm\frac{4\sqrt{3}}{7}\right)$

Key (C)

Sol.: $\frac{x^2}{16}+\frac{y^2}{4}=1$



Equation of normal at P

$\frac{4x}{\cos\theta}-\frac{2y}{\sin\theta} = 12$

If y = 0, x = 3cosθ ⇒ Q.(3cosθ, 0)

Let mid points PQ be M: (h, k)

⇒ 2h = 7cosθ, 2k = 2sinθ

⇒ locus of 'M'

$\frac{4x^2}{49}+\frac{y^2}{1}=1$ ... (i)

Eccentricity of ellipse is $\sqrt{3}/2$

Equation to Latus rectum x = ± ae ... (ii)

from (i) and (ii) we get y = ± 1/7

Hence the required point (± 2$\sqrt{3}$, ± 1/7)

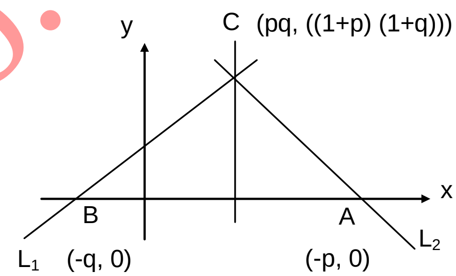
23. The locus of the orthocentre of the triangle formed by the lines (1 + p)x - py + p (1 + p) = 0, (1 + q) x - qy + q(1 + q) = 0, and y = 0, where p ≠ q, is

(A) a hyperbola (B) a parabola

(C) an ellipse (D) a straight line

Key (D)

Sol.: Let $L_1 \equiv \frac{x}{q}-\frac{y}{1+q} = 1$ –



$L_2 = \frac{x}{p}-\frac{y}{1+p} = -1$

Altitudes thro C is x = pq

and altitudes thro B is (1 + p) y + px + pq = 0

Eliminating p and q we get

y = -x so locus is a straight line

SECTION – II

Multiple Correct Answer Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

24.

24. If $I_n = \int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2, ... , then

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

Key (A, B, C)

24. $I_n = \int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2 ... (i)

$I_n = \int_{0}^{\pi}\frac{\sin nx}{\sin x}\left(\frac{1}{1+\pi^x}+\frac{1}{1\ +\pi^x}\right)dx$

$\Rightarrow I_n = \int_0^{\pi} \frac{\sin nx}{\sin x} dx$

$I_{n+2} - I_n = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx$

$= \int_0^{\pi} \frac{2\cos\left(\frac{nx+nx+2x}{2}\right).\sin\frac{nx \quad 2x \quad nx}{2}}{\sin x} dx$

$= \int_0^{\pi} \frac{2\cos(nx+x)\sin x}{\sin x} dx \;=\; 2\int_0^{\pi} \cos(n+1)x\, dx$

$= 2\frac{[\sin(n+1)x]_0^{\pi}}{n+1}$

$= 0$

$\therefore I_{n+2} = I_n \;\forall\; n = 0, 1, 2,$

$I_0 = 0$

$I_1 = \pi$

$\sum_{m=1}^{10} I_{2m+1} = 10\pi$

$\sum_{m=1}^{10} I_{2m} = 0$

***25.** An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(A) equation of ellipse is $x^2 + 2y^2 = 2$ (B) the foci of ellipse are $(\pm 1, 0)$

(C) equation of ellipse is $x^2 + 2y^2 = 4$ (D) the foci of ellipse are $(\pm \sqrt{2}, 0)$

Key (A, B)

25. $x^2 - y^2 = \frac{1}{2}$ (rectangular hyperbola)

eccentricity of rectangular hyperbola = $\sqrt{2}$

$\therefore$ eccentricity of ellipse = $\frac{1}{\sqrt{2}}$

Let equation of an ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{1}{\sqrt{2}} = \sqrt{1 - \frac{b^2}{a^2}}$

$\frac{b^2}{a^2} = \frac{1}{2}$ ... (1)

Let ellipse and hyperbola intersect at (α, β)

$\left(\frac{dy}{dx}\right)_{\text{at } (\alpha,\beta)}$ for hyperbola = $\left(\frac{\alpha}{\beta}\right)$

$\left(\frac{dy}{dx}\right)_{\text{at } (\alpha,\beta)}$ for ellipse = $-\frac{b^2}{a^2}\frac{\alpha}{\beta}$

$\therefore \left(\frac{\alpha}{\beta}\right)^2 \frac{b^2}{a^2} = 1$

$\Rightarrow \frac{\alpha^2}{\beta^2} = 2$ ... (ii)

$b^2/a^2 = 1/2$

As $\alpha^2 - \beta^2 = 1/2$

$\Rightarrow 2\beta^2 - \beta^2 = 1/2$

$\therefore \beta^2 = 1/2$

from (ii) $\alpha^2 = 1$ ... (iii)

Also $\frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$

$\Rightarrow \frac{1}{a^2} + \frac{1}{2b^2} = 1$

$\Rightarrow \frac{b^2}{a^2} + \frac{1}{2} = b^2$

$\Rightarrow \frac{1}{2} + \frac{1}{2} = b^2$

$\therefore b^2 = 1$

Also, $a^2 = 2$

$\therefore$ equation of an ellipse

$\frac{x^2}{2} + \frac{y^2}{1} = 1$

$\Rightarrow x^2 + 2y^2 = 2$ foci $\equiv (\pm 1, 0)$

26. For the function

$f(x) = x \cos \frac{1}{x}, x \geq 1,$

(A) for at least one x in the interval $[1, \infty)$, $f(x + 2) - f(x) < 2$

(B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x + 2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Key (B, C, D)

Sol.: $f(x) = x\cos \frac{1}{x}, x \geq 1$

$f'(x) = \cos \frac{1}{x} + \frac{1}{x} \sin \frac{1}{x}$ and $f''(x) = -\frac{1}{x^3} \cos \frac{1}{x}$

for $x \geq 1 \Rightarrow 0 < \frac{1}{x} \leq 1$

$\Rightarrow f''(x) < 0 \ \forall\ x \geq 1$

$\Rightarrow f'(x)$ is strictly decreasing in $[1, \infty)$

$\lim_{x\to\infty} f'(x) = \lim_{x\to\infty} \cos \frac{1}{x} + \frac{1}{x}\sin\frac{1}{x} = \cos 0 + 0 = 1$

Let $g(x) = f(x + 2) - f(x)$

$g'(x) = f'(x + 2) - f'(x) < 0$ (as $f'(x)$ is decreasing)

$\Rightarrow g(x)$ is decreasing

Now, $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} (x + 2) \cos \frac{1}{x+2} - x \cos \frac{1}{x}$

$= \lim_{x\to\infty} x\left(\cos\frac{1}{x+2} - \cos\frac{1}{x}\right) + 2\cos\frac{1}{x+2}$

$= \lim_{x\to\infty} 2x \sin\frac{x+1}{x^2+2x} . \sin\frac{1}{x^2+2x} + 2\cos\frac{1}{x+2}$

$$= \lim_{x\to\infty}\left[\frac{2(x^2+x)}{x^2+2x}\left(\frac{\sin\frac{x+1}{x^2+2x}}{\frac{x+1}{x^2+2x}}\right)\sin\frac{1}{x^2+2x}+2\cos\frac{1}{x+2}\right] = 2.1.1.\,0 + 2\,.1\ = 2$$

$\Rightarrow g(x) > 2\ \forall\ x \geq 1$

***27.** The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$

(B) directrix is $x = 0$

(C) latus rectum is $\frac{2a}{3}$

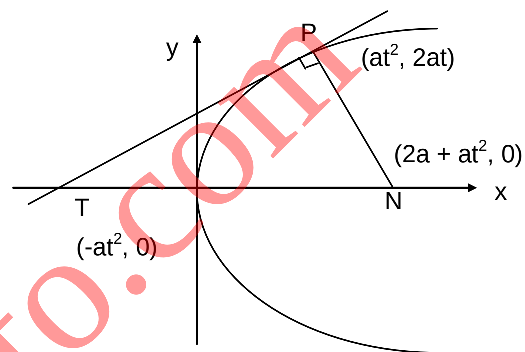
(D) focus is $(a, 0)$

Key (A, D)

Sol.: Tangent at P, $ty = x + at^2$

Normal at P, $y = -tx + 2at + at^3$

Let centroid be (h, k)



$h = \frac{2a + at^2}{3}$

$k = \frac{2at}{3}$

$\Rightarrow 3ah = 2a^2 + a^2t^2 = 2a^2 + \left(\frac{3k}{2}\right)^2$

$\Rightarrow 12ax = 8a^2 + 9y^2$

$\Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2a}{3}\right)$

Vertex: $\left(\frac{2a}{3}, 0\right)$

Latus rectum = $\frac{4a}{3}$

directrix: $x - \frac{2a}{3} + \frac{a}{3} = 0 \Rightarrow x = a/3$

Focus: $\left(\frac{2a}{3} + \frac{a}{3}, 0\right) = (a, 0)$

***28.** For $0 < \theta < \pi/2$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is/are

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{12}$

(D) $\frac{5\pi}{12}$

Key (C, D)

Sol.: $\sum_{m=1}^{6} \frac{1}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$

$$\Rightarrow \frac{1}{\sin\frac{\pi}{4}}\sum_{m=1}^{6}\frac{\sin\left(\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right)}{\sin\left(\theta + \frac{(m-1)\pi}{4}\right)\sin\left(\theta + \frac{m\pi}{4}\right)} = 4\sqrt{2}$$

$\Rightarrow \sum_{m=1}^{6}\left(\cot\left(\theta+\frac{(m-1)}{4}\right) - \cot\left(\theta+\frac{m}{4}\right)\right) = 4^{\pi}$ (π)

$\Rightarrow \cot\theta - \cot(\theta + \frac{3\pi}{2}) = 4$

$\Rightarrow \cot\theta + \tan\theta = 4$

$\Rightarrow \tan^2\theta - 4\tan\theta + 1 = 0$

$\Rightarrow \tan\theta = 2 \pm \sqrt{3} \Rightarrow \theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$

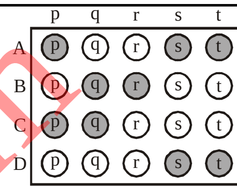
So, (C) and (D) are correct.

SECTION – III

Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:



29. Match the statements/expressions given in **Column I** with the values given in **Column II.**

	Column - I		**Column - II**
*(A)	Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p)	$\pi/6$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(q)	$\pi/4$
		(r)	$\pi/3$
(C)	Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(s)	$\pi/2$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$	(t)	π

Key **(A-q, s); (B-p, r, s, t); (C-t); (D-r)**

Sol.: (A) $\sin^2 2\theta = 2 - 2\sin^2\theta$

$\Rightarrow 4\sin^2\theta\cos^2\theta = 2\cos^2\theta$

$\Rightarrow 2\cos^2\theta(2\sin^2\theta - 1) = 0$

$\Rightarrow \cos\theta = 0$ or $\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{2}$ or $\frac{\pi}{4}$

(B) $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$

It is discontinuous when either

$\frac{6x}{\pi} \in I$ or $\frac{3x}{\pi} \in I$ i.e.,

when $x = \pi/6$ or $\pi/3$ or $\pi/2$ or π

(C) Volume = $|[\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}\ \ \hat{j}\ \ \hat{k}]|$ + π

$$= \left| \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} \right| = \pi$$

(D) $\vec{a} + \vec{b} + \sqrt{3}\vec{c} = 0$

$\Rightarrow |\vec{a} + \vec{b}|^2 = |\ \sqrt{3}\vec{c}\,|^2 -$

$\Rightarrow 1 + 1 + 2\cos\theta = 3$

$\Rightarrow \cos\theta = 1/2 \ \Rightarrow \theta = \pi/3$

30. Match the statements/expressions given in **Column I** with the values given in **Column II.**

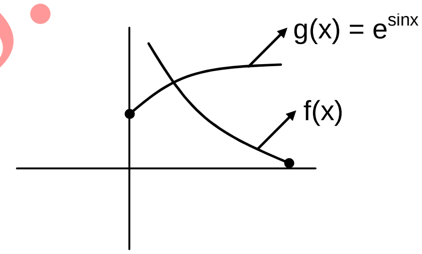
	Column - I		Column - II
(A)	The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p)	1
*(B)	Values(s) of k for which the planes kx + 4y + z = 0, 4x + ky + 2z = 0 and 2x + 2y + z = 0 intersect in a straight line	(q)	2
		(r)	3
(C)	Value(s) of k for which |x - 1| + |x - 2| + |x + 1| + |x +2| = 4k has integer solution(s)	(s)	4
(D)	If y′ = y + 1 and y(0) = 1, then values(s) of y (ln 2)	(t)	5

Key **(A-p), (B-q, s), (C-q, r, s, t), (D-r)**

Sol. (A) $x.e^{\sin x} - \cos x = 0$

$\Rightarrow e^{\sin x} = \dfrac{\cos x}{x}$

Let $f(x) = \dfrac{\cos x}{x}$, $g(x) = e^{\sin x}$



According the graph

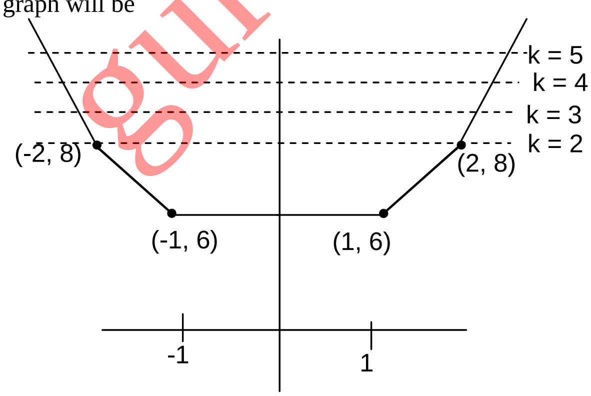
Number of solution is 1.

(B) $\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow k = 2, 4$

(C) |x + 1| + |x - 2| + |x + 1| + |x + 2| = 4k

graph will be



So solution is k = 2, 3, 4, 5

(D) $\dfrac{dy}{dx} = y + 1$

$$\int \frac{dy}{y+1} = \int dx$$

ln (y + 1) = x + C

(x + c)

$y + 1 = e^{(x+c)}$

$y = c^{x+c} - 1$

x = 0, y = 1

$1 = e^c - 1$

$e^c = 2$

C = ln 2

$y = e^{x + \ln 2} - 1$

for x = ln2

$e^{2\ln 2} - 1$

y = 3.

SECTION – IV

Integer Answer Type

This section contains 8 questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:

31. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \le 9x\}$ is

Key 7

Sol.: $A = \{x \mid x^2 + 20 - 9x \le 0\}$

$x^2 - 9x + 20 \le 0$

$(x - 5)(x - 4) \le 0$

$x \in [4, 5]$

$f(x) = 2x^3 - 15x^2 + 36x - 48$

$f'(x) = 6(x - 2)(x - 3)$

sign scheme of $f'(x)$

\+ 2 – 3 +

f(x) is strictly increasing in (4, 5)

So, f(5) = 7

32. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations:

3x - y - z = 0

-3x + z = 0

-3x + 2y + z = 0

Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is

Key 7

Sol.:

$3x - y - z = 0$ …..(i)

$-3x + z = 0$ …..(ii)

$-3x + 2y + z = 0$ …..(iii)

Solving (i) & (ii)

y = 0

So 3x – z = 0

z = 3x

Now $x^2 + y^2 + z^2 \le 100$

$x^2 + 9x^2 \le 100$

$|x| \le \sqrt{10}$

$-\sqrt{10} \le x \quad \sqrt{10} \quad \le$

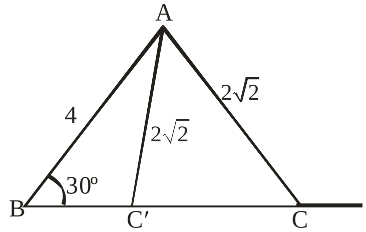
Integral values of ‘x’ are –3, –2, –1, 0, 1, 2, 3

So ‘7’ points are there.

***33.** Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is

Key 4

Sol.:



Using sine rule in ΔABC

$$\frac{\sin C}{4} = \frac{\sin 30°}{2\sqrt{2}} \Rightarrow C \quad 45 \quad = \quad °$$

∴ ∠AC′C = 45º and ∠C′AC = 90º

Difference of area of ΔABC & ΔABC′ is area of ΔACC′

$$= \frac{1}{2}(2\sqrt{2})(2\sqrt{2}) = 4$$

34. Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right)$ = 2. Then the value of p(2) is

Key 0

Sol.: Let P(x) = $ax^4 + bx^3 + cx^2 + dx + e$

Given $\lim_{x\to 0}\left(1+\frac{P(x)}{x^2}\right) = 2$

Limit exist only if, d = e = 0

$$\lim_{x\to 0}[1 + ax^2 + bx \quad c] \quad 2 \quad + \quad =$$

⇒ c + 1 = 2

⇒ c = 1

P(x) = $ax^4 + bx^3 + x^2$

P′(x) = $4ax^3 + 3bx^2 + 2x$

= $x(4ax^2 + 3bx + 2)$

Note: $4ax^2 + 3bx + 2 \equiv \lambda(x - 1)(x - 2) = \lambda(x^2 - 3x + 2)$

$\Rightarrow \lambda = 1,\ a = \frac{1}{4},\ b = -1$

$\Rightarrow \quad P(x) = \frac{1}{4}x^4 - x^3 \quad x^2 \quad +$

$\therefore \quad P(2) = \frac{1}{4}2^4 - 2^3 + 2^2$

= 4 – 8 + 4 = 0

35. Let f : R → R be a continuous function which satisfies

f(x) = $\int_0^x f(t)\,dt$. Then the value of f(ln 5) is

Key 0

Sol.: f ′(x) = f(x)

⇒ f(x) = ce^x

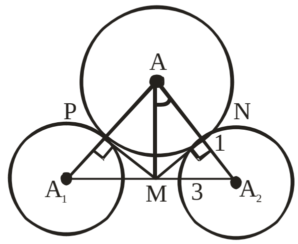
⇒ c = 0 because f(0) = 0

∴ f(ln 5) = 0

***36.** The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 extrenally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

Key 8

Sol.:



$PM = PN = \sqrt{9\quad 1}\quad 2\sqrt{2}-\qquad =$

Clearly AM perpendicular to A_1A_2

Let radius of circle C = r

In ΔAMA_2

$(AM)^2 + 9 = (r + 1)^2$ …..(i)

In ΔAMN

$(AM)^2 = r^2 + 8$

from (i) and (ii) r = 8.

***37.** The smallest value of k, fow which both the roots of the equation

$x^2 - 8kx + 16(k^2 - k + 1) = 0$

are real, distinct and have values at least 4, is

Key 2

Sol.: For the root to be real & distinct and having minimum value, following points should hold :

(1) $D > 0$

(2) $-\frac{b}{2a} > 4$

(3) $f(4) \ge 0$

(1) $64k^2 - 64(k^2 - k + 1) > 0$

$\Rightarrow k > 1$

(2) $\frac{8k}{2} > 4 \Rightarrow k \quad 1 \quad >$

(3) $16 - 32k + 16k^2 - 16k + 16 \ge 0$

$(k - 2)(k - 1) \ge 0$

$k \ge 2$ or $k \le 1$

Taking the intersection the required solution is $k \ge 2$.

Alternate

$(x - 4k)^2 = 16(k - 1)$

$\Rightarrow k \ge 1$

$\Rightarrow x = 4k \pm 4\sqrt{k-1}$

real solutions for $k \ge 1$

Note : $4k + 4\sqrt{k-1} \ge 4$ always true for $k \ge 1$

Now, $4k - 4\sqrt{k-1} \ge 4$

$\Rightarrow k - 1 \ge \sqrt{k-1}$

$\Rightarrow (k - 1)^2 \ge (k - 1)$

$\Rightarrow (k - 1)(k - 2) \ge 0$

$\Rightarrow k \geq 2$ for distinct & both roots more than 4.

38. If the function $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

Key 2

Sol.: $f(x) = x^3 + e^{x/2}$

$g(x) = f^{-1}(x)$

$f(g(x)) = x \quad f(0) = 1 \Rightarrow f^{-1}(1) = 0$

$\Rightarrow g(1) = 0$

$$f'(g(x)).g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(1) = \frac{1}{f'(g(1))}$$

$$\Rightarrow \quad = \frac{1}{f'(0)}$$

$$f'(x) = 3x^2 + \frac{1}{2}e^{x/2}$$

$$f'(0) = \frac{1}{2}$$

$$g'(1) = 2$$

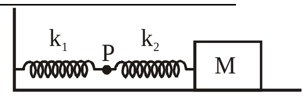
PART – III: PHYSICS

SECTION – I

Straight Objective Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

*39. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is



(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$

(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$.

Key, (D)

Sol. $x_1 + x_2 = A$

$k_1 x_1 = k_2 x_2$

$\therefore$ amplitude of point $P = \frac{k_2 A}{k_1 + k_2}$.

$\therefore$ (D) is correct.

*40. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y–axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x–axis with constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, form the y–axis is

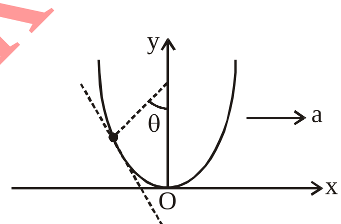
(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$

(C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$.

Key (B)

Sol. $\tan\theta = \frac{a}{g}$

slope of tangent

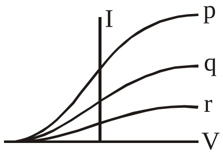


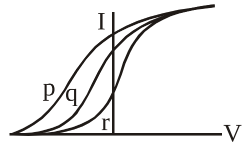
$= \tan(\pi - \theta) = \tan\theta = \frac{dy}{dx} = 2kx$

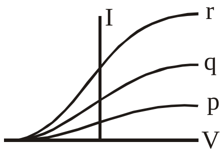
$\Rightarrow \quad x = \frac{a}{2kg}$ –

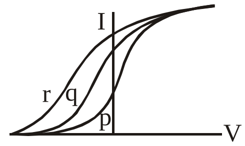
$\therefore$ (B) is correct.

41. Photoelectric effect experiments are performed using three different metal plates p, q and r having work function $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelength of 550 nm and 350 nm with equal intensities illuminates each of the plates. The correct I–V graph for the experiment is

(A) 

(B) 

(C) 

(D) 

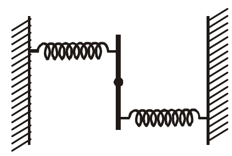
Key. (A)

Sol. The work function for P is smallest.

∴ stopping potential for P is largest. Secondly, not all wavelengths will be able to eject photoelectron from all three i.e., the saturation current will be different.

∴ (A) is correct.

*42. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is

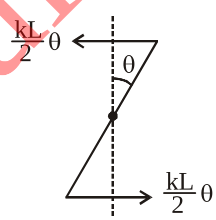


(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$.

Key. (C)

Sol. Restoring torque $\tau_r = k\frac{L}{2}\frac{L}{2}\theta\, 2 \cdot \times$



$\Rightarrow \frac{ML^2}{12}\alpha = \frac{k}{2}L^2\theta \Rightarrow \alpha = -\frac{6k}{M}\theta$

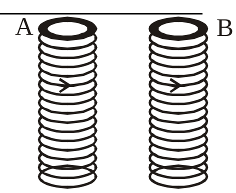
∴ $f = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ ∴ (C) is correct.

SECTION – II

Multiple Correct Answer Type

This section contains 5 multiple correct answer(s) type questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

43. Two metallic rings A and B, identical in shape and size but having different resistivity ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is (are)



(A) $\rho_A > \rho_B$ and $m_A = m_B$ (B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$ (D) $\rho_A < \rho_B$ and $m_A < m_B$

Key. (B, D)

Sol. $q = \dfrac{\Delta\phi}{R} \propto \dfrac{1}{\rho}$ …(i)

$\int I\ell B_\gamma dt = mv$

$\Rightarrow \quad \ell B_\gamma q = mv$

$\Rightarrow \quad v \propto \dfrac{q}{m}$ …(ii)

Also $v^2 \propto h$ …(iii)

From (i), (ii) and (iii)

$(m\rho)_A < (m\ \ )_B\ \ \rho$

$\therefore$ (B) and (D) are correct.

*44. A student performed the experiment to measure the speed of sound in air using resonance air–column method. Two resonances in the air–column were obtained by lowering the water level. The resonance with the shorter air–column is the first resonance and that with the longer air–column is the second resonance. Then,

(A) the intensity of the sound heard at the first resonance was more than that at the second resonance
(B) the prongs of the tunning fork were kept in a horizontal plane above the resonance tube
(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm
(D) the length of the air–column at the first resonance was somewhat shorter than $1/4^{th}$ of the wavelength of the sound in air.

Key. (A, D)

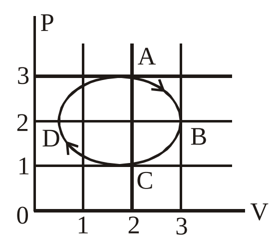
Sol. As length of air–column increases intensity decreases.

Hence (A) is correct.

$\ell + e = \dfrac{\lambda}{4} \Rightarrow \ell < \dfrac{\lambda}{4}$.

Hence (D) is correct.

*45. The figure shows the P–V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi–circle and CDA is half of an ellipse. Then,



(A) the process during the path $A \rightarrow B$ is isothermal
(B) heat flows out of the gas during then path $B \rightarrow C \rightarrow D$
(C) work done during the path $A \rightarrow B \rightarrow C$ is zero
(D) positive work is done by the gas in the cycle ABCDA.

Key. (B, D)

Sol. Temperature at B > temperature at D

$\therefore$ ΔU is negative (for $B \rightarrow C \rightarrow D$)

Also W is negative (for $B \rightarrow C \rightarrow D$)

Tracing is clockwise on PV diagram.

$\therefore$ W is positive.

$\therefore$ (B) and (D) are correct.

46. Under the influence of the Coulomb field of charge +Q, a charge –q is moving around it in an elliptical orbit. Find out the correct statement (s)

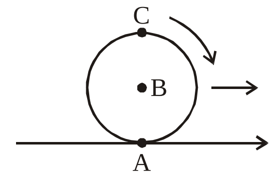
(A) the angular momentum of the charge –q is constant
(B) the linear momentum of the charge –q is constant
(C) the angular velocity of the charge –q is constant
(D) the linear speed of the charge –q is constant.

Key. (A)

Sol. Force is central.

So, option (A) is correct.

*47. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

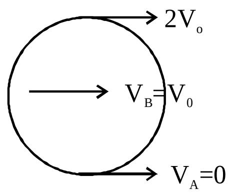


(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ (B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$ (D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$.

Key. (B, C)

Sol.



$\vec{V}_B - \vec{V}_A = \vec{V}_o$

$\vec{V}_C - \vec{V}_B = \vec{V}_o$

$|\vec{V}_C - \vec{V}_A| = 2V_o$

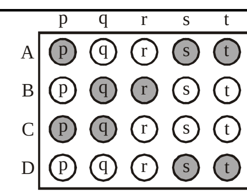
$|\vec{V}_C - \vec{V}_B| = V_o$

So, options (B) and (C) are correct.

SECTION – III
Matrix Match Type

*This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labeled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:*

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following:

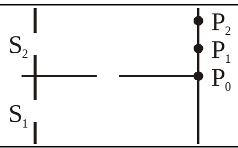
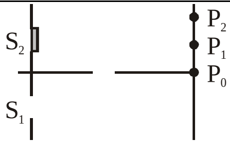
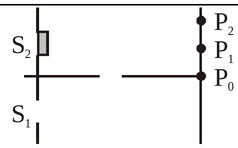
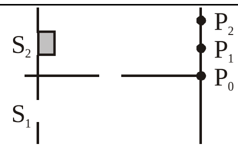


48. Column II gives certain systems undergoing a process. Column I suggests changes in some of the parameters related to the system. Match the statements in Column I to the appropriate process(es) from Column II.

	Column I		Column II	
(A)	The energy of the system is increased	(p)	System : Process :	A capacitor, initially uncharged. It is connected to a battery.
(B)	Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q)	System : Process :	A gas in an adiabatic container fitted with an adiabatic piston. The gas is compressed by pushing the piston.
(C)	Internal energy of the system is converted into its mechanical	(r)	System : Process :	A gas in a rigid container. A gas gets cooled due to colder atmosphere surrounding it.
(D)	Mass of the system is decreased	(s)	System : Process :	A heavy nucleus, initially at rest. The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted.
		(t)	System : Process :	A resistive wire loop. The loop is placed in a time varying magnetic field perpendicular to its plane.

Key. (A) – (p, q, t) [When current will pass through loop its temperature will increase]
(B) – (q)
(C) – (s)
(D) – (s)

49. Column I shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 = S_2P_1 = \lambda/2$ and $S_1P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in Column I with the statement(s) in Column II valid for that situation.

	Column I		**Column II**
(A)		(p)	$\delta(P_0) = 0$
(B)	 $(\mu-1)t = \lambda/4$	(q)	$\delta(P_1) = 0$
(C)	 $(\mu-1)t = \lambda/2$	(r)	$I(P_1) = 0$
(D)	 $(\mu-1)t = 3\lambda/4$	(s)	$I(P_0) > I(P_1)$
		(t)	$I(P_2) > I(P_1)$

Key. (A) – (p, s), (B) – (q), (C) – (t), (D) – (r, s, t)

Sol. (A) – (p, s)

$$I(P_1) = I_{max}\cos^2\frac{\pi}{4}$$

$$I(P_2) = I_{max}\cos^2\frac{\pi}{3}$$

(B) – (q)
P_1 is central maxima

(C) – (t)
P_o is minima in this case
and At, P_2

path difference is $\frac{\lambda}{6}$ $\left(\phi = \frac{\pi}{3}\right)$

while at P_1 path difference is $\frac{\lambda}{4}$ $\left(\phi = \frac{\pi}{2}\right)$.

(D) – (r, s)

At P_o $\Delta x = \frac{3x}{4} \Rightarrow \phi = \frac{3\pi}{2}$

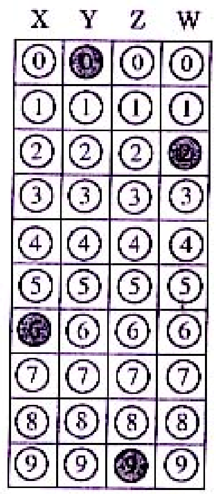
At P_1 $\Delta x = \frac{x}{2} \Rightarrow I(P_1) = 0$

At P_2 $\Delta x = \frac{15x}{12} \Rightarrow \phi = \frac{5\pi}{6}$

SECTION – IV

Integer Answer Type

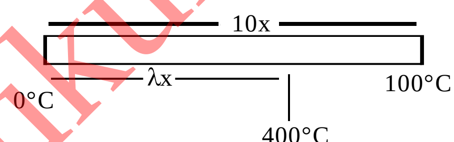
This section contains 8 questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



*50. A metal rod AB of length 10x has its one end A in ice at 0ºC and the other end B in water at 100ºC. If a point P on the rod is maintained at 400ºC, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ.
[Neglect any heat loss to the surrounding.]

Key. 9.

Sol. $\left(\frac{\Delta Q}{\Delta t}\right)_{\ell} = \frac{300}{\frac{10x-\lambda x}{KA}} = mL_V$ …(1)



and $\left(\frac{\Delta Q}{\Delta t}\right)_{w} = \frac{400}{\frac{\lambda x}{KA}} = mL_f$ …(2)

dividing $\frac{300}{10x-\lambda x}\times\frac{\lambda x}{400} = \frac{L_v}{L_f}$

$\Rightarrow \frac{3}{4}\frac{\lambda}{(10-\lambda)} = \frac{540}{80}$

$\lambda = 9(10-\lambda)$

$10\lambda = 90$

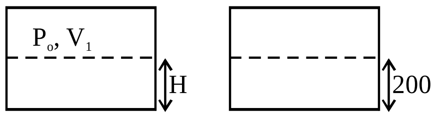
$\Rightarrow \lambda = 9$

*51. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel become steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure = 1.0×10^5 N/m^2, density of water = 1000 kg/m^3 and g = 10 m/s^2. Neglect any effect of surface tension.]

Key. 6 mm.

Sol.



$P_1 V_1 = P_2 V_2$

$P_o \times A(0.5 - H) \quad (P_o \quad g \quad 0.2)A \quad 0.3 \quad - \quad \rho \quad \times \quad \times$

$\Rightarrow \; 10^5(0.5 - H) \quad (10^5 \quad 2 \quad 10^3) \quad 0.3 \quad - \quad \times \quad \times$

$\Rightarrow \; 100(0.5 - H) \quad (100 \quad 2) \quad 0.3 \quad = \quad - \quad \times$

$$0.5 - H = \frac{29.4}{100}$$

$\Rightarrow$ H = 0.5 - .294

$\Rightarrow$ H = 0.206

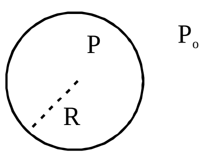
$\Rightarrow$ H = 206 mm

So fall in height = 6 mm.

*52. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap–water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

Key. 6.

Sol.



then $P = P_o + \frac{4S}{r}$

Now $P \times \frac{4}{3}\pi r^3 \quad nR_g T$

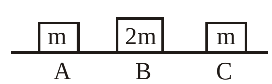
$\Rightarrow \; \left(P_o + \frac{4S}{r}\right)\frac{4}{3}\pi r^3 \quad nR_g T \quad =$

For two bubbles

$$\frac{\left(P_o + \frac{4S}{r_A}\right) r_A^3}{\left(P_o + \frac{4S}{r_B}\right) r_B^3} = \frac{n_A}{n_B}$$

$$\frac{\left(8 + \frac{4 \times 0.04}{2 \times 10^{-2}}\right)\left(2 \times 10^{-2}\right)^3}{\left(8 + \frac{4 \times 0.04}{4 \times 10^{-2}}\right)\left(4 \times 10^{-2}\right)^3} = \frac{n_A}{n_B} \quad \Rightarrow \quad \frac{n_B}{n_A} = 6$$

*53. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.



Key. 4 m/s.

Sol. For collision between A and B

$m\times 9+2m\quad 0\times mV_1 = 2mV_2+$

$\Rightarrow \quad 9 = V_1 + 2V_2$ …(1)

$e = 1$

So

$9 = V_2 - V_1$ …(2)

Solving (1) and (2)

$V_2 = 6$ m/s

For collision between B and D

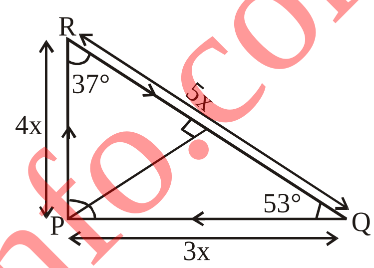
$2m\times 6=(2m\quad m)v$

$\Rightarrow \quad v=\frac{2\times 6}{3} = 4$ m/s

54. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = 3x, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

Key. K = 7.

Sol. d = 4x cos 37



$= 4x\times\frac{3}{5}$

Now,

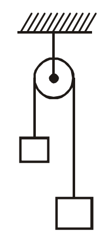
$B_P = \frac{\mu_o I}{4\pi d}[\sin 37+\sin 53]$

$= \frac{\mu_o I}{4\pi\frac{12x}{5}}\left[\frac{7}{5}\right] = \frac{7}{48}\frac{\mu_o I}{\pi x}$

$= 7\left(\frac{\mu_o I}{48\pi x}\right)$

So, K = 7

*55. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two block of masses 0.36 and 0.72 kg. Taking g = 10 m/s^2, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



Key. 8 J.

Sol. $a = \frac{(m_2-m_1)g}{m_1+m_2} = \frac{0.36\times 10}{1.08}\quad \frac{10}{3} m/s^2$ =

Displacement

$S=\frac{1}{2}gt^2$

$=\frac{1}{2}\times\frac{10}{3}\quad (1)^2\times\quad\frac{5}{3}m\ =$

$T=\frac{2m_1m_2g}{m_1+m_2}=\frac{2\times 0.36\times 0.72\quad 10}{3\times 0.36}N\quad\times$

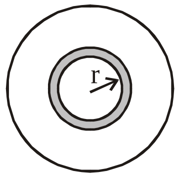
$=2\times 2.4N\quad 4.8N$

$W = \vec{T}.\vec{S} \quad 1.6 \ \frac{5}{3} \ \times 4.8 = \frac{5}{3} \quad 8J \ \times \ =$

56. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = \kappa r^a$, where κ and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at r = R, find the value of a.

Key. 2.

Sol. for the element



$dq = \rho dV$

$= kr^{\alpha} 4\pi r^2 dr$

$q = \int dq = \int kr^{\alpha} 4 \ r^2 dr \qquad \pi$

$q = \frac{k4\pi r^{\alpha+3}}{\alpha+3}$

$E_r = \left[\frac{K4\pi r^{\alpha+3}}{\alpha+3}\right]\frac{1}{r^2}$

$E_R = \left[\frac{K4\pi r^{\alpha+3}}{\alpha+3}\right]\frac{1}{R^2}, \quad \frac{E_r}{E_R} = \frac{1}{8} = \frac{r^{\alpha+1}}{R^{\alpha+1}}$ (where r = R/2)

$\Rightarrow \quad \alpha = 2$

*57. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

Key. 5 cm.

Sol. $\ell = 20m$, $m = 1$ gm, $T = 0.5$ N

f = 100 Hz

$\mu = \frac{1\times10^{-3}}{20\times10^{-2}} = \frac{1}{2} \times 10^{-2}$

$V = \sqrt{\frac{0.5}{0.5\times10^{-2}}} = 10 m/s, \quad \lambda = 0.1$

$\frac{\lambda}{2} = 0.05m = 5cm$.

MARKING SCHEME

PAPER – I

1. For each questions in **Section I**, you will be **awarded 3 marks** if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In case of bubbling of incorrect answer, **minus one (–1) mark** will be awarded
2. For each question in **Section II**, you will be **awarded 4 marks** if you have darkened all the bubble(s) corresponding to the correct choice(s) for the answer, and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
3. For each question in **Section III**, you will be **awarded 4 marks** if you darken the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
4. For each question in **Section IV**, you will be **awarded 2 marks** for **each row** in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of 8 marks. There is **no negative marking** for incorrect answer(s) for this section.

PAPER – II

1. For each question in **Section – I**, you will be awarded 3 marks if you have darkened only the bubble corresponding to the correct answer and **zero mark** if no bubble is darkened. In case of bubbling of incorrect answer, **minus one (–1) mark** will be awarded.
2. For each question in **Section II**, you will be **awarded 4 marks s** if you have darkened all the bubble(s) corresponding to the correct answer and **zero mark** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
3. For each question in **Section III**, you will be **awarded 2** for each row in which you have darkened the bubble(s) corresponding to the correct answer. Thus, each question in this section carries a maximum of 8 marks. There is no negative marking for incorrect answer(s) for this section.
4. For each question in **Section IV**, you will be **awarded 4 marks** if you darken the bubble corresponding to the correct answer, and zero mark if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.