



ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER - 2007

DISCRETE MATHEMATICAL STRUCTURE

SEMESTER - 1

Time : 3 Hours]

[Full Marks : 70

GROUP - A

(Multiple Choice Type Questions)

Choose the correct alternatives for any ten of the following :

10 × 1 = 10

i) Null set is the subset of

a) universal set

b) universe of discourse

c) every set

d) none of these.

ii) Cardinality of the power set of a non-empty set A is

a) $2^{|A|}$

b) $2 | A |$

c) $| A |^2$

d) none of these.

iii) A partial order relation is

a) always antisymmetric

b) sometimes antisymmetric

c) irreflexive

d) none of these.

iv) Degrees contributed by a loop to a vertex in a graph G is

a) 2

b) 1

c) 0

d) none of these.

v) A simple graph

a) does not possess a loop

b) must possess a loop

c) is necessarily a multigraph

d) is necessarily a pseudograph.

vi) The relation \subset is

a) irreflexive

b) antisymmetric

c) asymmetric

d) all of these.



vii) If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are injective, then $g \circ f$ is

- a) injective b) surjective
c) invertible d) none of these.

viii) $\{a, b\} \leq V_T$ and $S \in V_N$, then $S \rightarrow ab$ is a

- a) type-0 grammar b) type-1 grammar
c) type-2 grammar d) type-3 grammar.

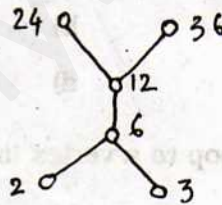
ix) Minimum height of an n -vertex binary tree is

- a) $\frac{n-1}{2}$ b) $\frac{n+1}{2}$
c) $\lceil \log_2(n+1) - 1 \rceil$ d) $\lfloor \log_2(n+1) - 1 \rfloor$.

x) Let A be a set with 10 distinct elements. How many different binary relations on A are there ?

- a) 100 b) 100^2
c) 2^{100} d) none of these.

xi) Hasse diagram is given below :



This is a

- a) Poset b) Toset
c) Lattice d) none of these.

xii) Six boys and four girls can sit in a row in

- a) $6! \times 4!$ ways b) $2 \times 6! \times 4!$ ways
c) 2^{24} ways d) none of these.



xiii) Let $|A| = |B| = n$, where $|A|$ denotes number of elements of A . Then the number of bijective mappings from A to B is

- a) 1
 b) n^n
 c) $n!$
 d) can't be said.

xiv) Let $f: Z \rightarrow Z$ given by $f(x) = 2x$, $x \in Z$. Then

- a) f is injective but not surjective
 b) f is surjective but not injective
 c) f is bijective
 d) none of these.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following.

$3 \times 5 = 15$

2. Define distributive lattice. Prove that in a distributive lattice

$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a).$$

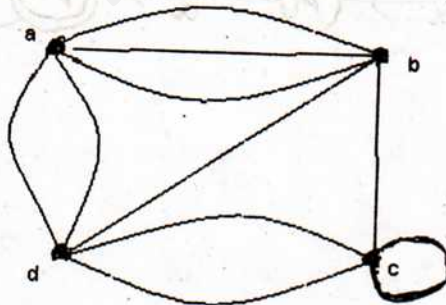
3. Prove that number of vertices of odd degree in a graph $G = (V, E)$ is always even.

4. Find the particular solution of the difference equation :

$$a_{r+2} - 5a_{r+1} + 6a_r = 5^r$$

5. Prove that the number of circular permutations of n different objects is $(n-1)!$

6. Define adjacency matrix of a simple graph $G = (V, E)$. Write down the adjacency matrix for the following undirected graph :



Use mathematical induction to prove that $(3^n + 7^n - 2)$ is divisible by 8,

here $n \geq 1$.



GROUP - C

(Long Answer Type Questions)

Answer any three questions.

 $3 \times 15 = 45$

8. a) Determine the union of the following two fuzzy sets :

$$A = \left\{ \frac{4}{0.1}, \frac{6}{0.5}, \frac{8}{0.6}, \frac{10}{0.7} \right\} \text{ and } B = \left\{ \frac{0}{0.4}, \frac{2}{0.6}, \frac{4}{1}, \frac{6}{1}, \frac{8}{0.6}, \frac{10}{0.5} \right\}.$$

- b) Show that the mapping $f: R - \{\sqrt{2}\} \rightarrow R$ defined by $f(x) = \frac{x}{x^2 - 2}$, $x \neq \sqrt{2}$ is surjective but not injective.

- c) A simple graph with n vertices and k components can have at most

$$(n - k)(n - k + 1) / 2 \text{ edges.}$$

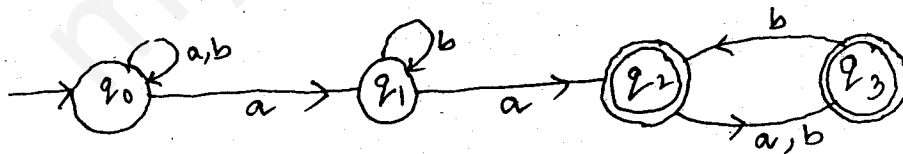
3 + 6 + 6

9. a) Design an FA which accepts the language

$L = \{ w/w \text{ has both an even number of 0's and even number of 1's over alphabet } \Sigma = \{0, 1\} \}$.

- b) Design an FA which accepts set of strings containing exactly four 1's in every string over alphabet $\Sigma = \{0, 1\}$.

- c) Convert the following NFA into DFA.



5 + 4 +

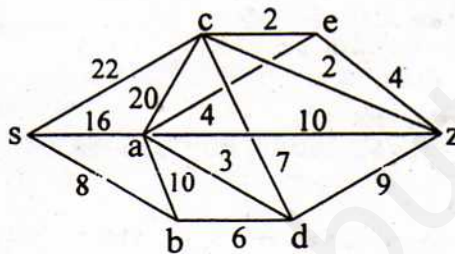
10. a) Given the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ and the initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$, find the solution.

- b) Find solution for the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 6$.

8 +



11. a) If $P(S)$ is the power set of a set S and \cup and \cap are taken as the join and meet, prove that $\{P(S), \subseteq\}$ is a lattice.
- b) Define partially ordered set. Can you say all partially ordered sets are lattice? Justify.
- c) Give an example of a relation ρ on A (described by you) which is symmetric and transitive but not reflexive with justification. 5 + 6 + 4
12. a) State Dijkstra's algorithm for shortest path problem.
- b) Use Dijkstra's algorithm to find the shortest path between the vertices from s to z in the following graph.



5 + 10

13. Write short notes on any *three* of the following :

3 × 5

- a) Bipartite graph
- b) DFA
- c) NFA
- d) Moore machine
- e) Mealy machine.

END