ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2008

DISCRETE MATHEMATICAL STRUCTURE

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

Graph sheet is provided on Page 31.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) In a group of 400 people, 250 can speak in English only, 70 can speak Hindi only.

How many can speak in English ?

a) 250 b) 330

c) 400 d) 320.

ii) If the general term of the sequence $\{a_k\}$ be a^k, which will be the generating function ?

a) $1/(1-x)$ b) $a/(1-x)$

c) $k/(1-x)$ d) $1/(1-ax)$.

iii) A simple graph with n vertices has maximum

a) $n(n-1)/2$ edges b) $(n-1)$ edges

c) $n(n+1)/2$ edges d) n^2 edges.

iv) If n be the number of vertices, e be the number of edges and k be the number of components of a graph G, then

a) $e > n + k$ b) $e \geq n - k$

c) $e \leq n - k$ d) none of these.

v) $A \cap B^c =$

a) $A - B$

b) $(A \cup B)^c$

c) $A - B^c$

d) none of these.

vi) If $A = \{1, 2, 3\}$, $B = \{4, 5\}$, $C = \{1, 2, 3, 4, 5\}$, then $(C \times B) - (A \times B) =$

a) $(C - A) \times (B - A)$

b) $B \times B$

c) $(C \cap A) \times B$

d) none of these.

vii) If A and B are two fuzzy sets given by

$A = \{(1, 0{\cdot}1), (3, 0{\cdot}4), (5, 0{\cdot}2), (7, 0{\cdot}8)\}$ and

$B = \{(1, 0{\cdot}3), (3, 0{\cdot}2), (5, 0{\cdot}5), (7, 0{\cdot}7)\}$ then

a) $A \cup B = \{(1, 0{\cdot}3), (3, 0{\cdot}4), (5, 0{\cdot}2), (7, 0{\cdot}8)\}$

b) $A = \{(1, 0{\cdot}1), (3, 0{\cdot}4), (5, 0{\cdot}5), (7, 0{\cdot}8)\}$

c) $A \cup B = \{(1, 0{\cdot}3), (3, 0{\cdot}4), (5, 0{\cdot}5), (7, 0{\cdot}8)\}$

d) none of these.

viii) If the function $f: R \rightarrow R$ defined by

$$f(x) = \begin{cases} 3x - 4, & x > 0 \\ -3x + 2, & x \leq 0 \end{cases}$$

then $f^{-1}(2) =$

a) $\{2\}$

b) $\{0, 2\}$

c) $\{2, -2\}$

d) none of these.

ix) The generating function of the sequence $\{0, 1, 0, -1, 0, 1, 0, -1, 0, \ldots\}$ is

a) $\dfrac{1}{1 + x^2}$

b) $\dfrac{x}{1 + x^2}$

c) $\dfrac{x^2}{1 + x^2}$

d) none of these.

x) A complete graph of n vertices has exactly

a) $\frac{n(n+1)}{2}$ vertices
b) $\frac{n(n-1)}{2}$ vertices
c) $\frac{(n+1)}{2}$ vertices
d) none of these.

xi) Cardinality of the power set of a non-empty set A is

a) $2^{|A|}$
b) $2|A|$
c) $|A|^2$
d) none of these.

xii) The solution of the recurrence relation

$$a_r - 7a_{r-1} + 10a_{r-2} = 0 \text{ given } a_0 = 0,\ a_1 = 3 \text{ is}$$

a) $a_r = 5^{-r} - 2^r$
b) $5^r + 2^r$
c) $5^r - 2^r$
d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the following using generating function :

$$a_n - a_{n-1} = 3(n-1),\ n \geq 1, \text{ and where } a_0 = 2.$$

3. Find the coefficient of x^{18} in $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + x^5 + \dots)^5$.

4. Let A be some fixed 10-element subset of $S = \{1, 2, 3, 4, 5, \dots 50\}$. Show that A possesses two different 5-element subsets, the sums of whose elements are equal.

5. Show that $4^{2n+1} + 3^{n+2}$ is an integer multiple of 13, for all positive integers n.

6. Draw the graph represented by the given adjacency matrix :

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

v) $A \cap B^c =$

a) $A - B$ b) $(A \cup B)^c$

c) $A - B^c$ d) none of these.

vi) If $A = \{1, 2, 3\}$, $B = \{4, 5\}$, $C = \{1, 2, 3, 4, 5\}$, then $(C \times B) - (A \times B) =$

a) $(C - A) \times (B - A)$ b) $B \times B$

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a) $A \cup B = \{(1, 0{\cdot}3), (3, 0{\cdot}4), (5, 0{\cdot}2), (7, 0{\cdot}8)\}$

b) $A = \{(1, 0{\cdot}1), (3, 0{\cdot}4), (5, 0{\cdot}5), (7, 0{\cdot}8)\}$

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d) none of these.

viii) If the function $f : R \to R$ defined by

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then $f^{-1}(2) =$

a) $\{2\}$ b) $\{0, 2\}$

c) $\{2, -2\}$ d) none of these.

ix) The generating function of the sequence $\{0, 1, 0, -1, 0, 1, 0, -1, 0, \ldots\}$ is

a) $\dfrac{1}{1 + x^2}$ b) $\dfrac{x}{1 + x^2}$

c) $\dfrac{x^2}{1 + x^2}$ d) none of these.

x) A complete graph of n vertices has exactly

a) $\frac{n(n+1)}{2}$ vertices

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GROUP – B

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4. Let A be some fixed 10-element subset of $S = \{1, 2, 3, 4, 5, \dots 50\}$. Show that A possesses two different 5-element subsets, the sums of whose elements are equal.

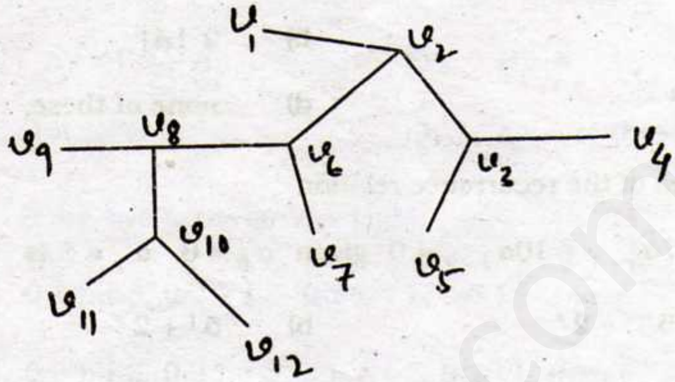
5. Show that $4^{2n+1} + 3^{n+2}$ is an integer multiple of 13, for all positive integers n.

6. Draw the graph represented by the given adjacency matrix :

$$\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

7. Find the generating function for the sequnce 1 1 0 1 1 1 1 ....

8. Explain the Ring Sum operation with an example. Find the centre of the following graph :



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions. 3 × 15 = 45

9. Let R and S be two fuzzy relations from X to Y given in the following matrix forms. Find (a) $R \cup S$, (b) $R \cap S$, (c) $R + S$ and (d) $R \cdot S$.

$$M_R = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} 0\cdot3 & 1 & 0\cdot2 \\ 0\cdot8 & 0 & 0\cdot5 \end{pmatrix} \end{array}$$

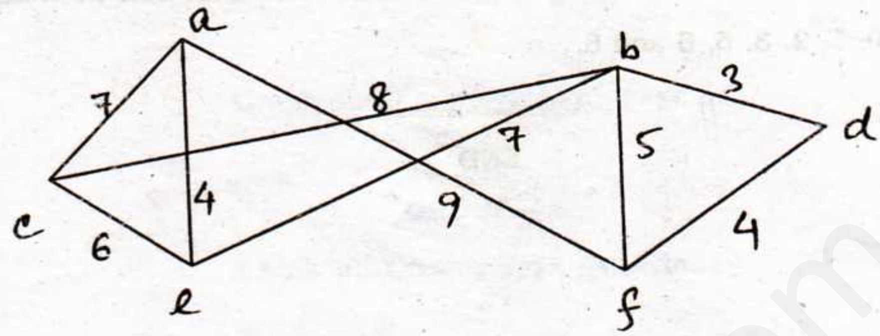
$$M_S = \begin{array}{c} \\ x_1 \\ x_2 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{pmatrix} 0\cdot6 & 0\cdot1 & 0\cdot9 \\ 0 & 0\cdot2 & 0\cdot3 \end{pmatrix} \end{array}$$

Draw Hasse-diagram to illustrate the following partial ordering :

The set of all subsets of { 1, 2, 3, 4 } having at least two numbers partially ordered by $\subseteq$. Show that $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + 1/2 \rfloor$ where x is a real number. 8 + 5 + 2

10. Prove that s simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges. Prove that in a tree there exists one and only one path between every pair of vertices. 6 + 9

11. Find the shortest path of the following graph using Prim's algorithm :



Given the post-order and inorder traversals of a binary tree. Draw the unique binary tree :

Post-order : d e c f b h i g a

Inorder : d c e b f a h g i 8 + 7

12. a) Define grammar of a language and its types. Give an example of a grammar which is Type 2 but not Type 3. 2 + 3

b) Find the grammar for the language

$$L = L = \{ w \in \{a, b, c\}^* : w = a^n b^n c^m, n \geq 1, m \geq 0 \}.$$ 5

c) Define Mealy machine and Moore Machine. Construct a Moore machine from the following Mealy machine : 5

Present State	**Next State**			
	a = 0		**a = 1**	
	State	Output	State	Output
s_0	s_0	1	s_1	0
s_1	s_3	1	s_3	1
s_2	s_1	1	s_2	1
s_3	s_2	0	s_0	1

13. a) Define a lattice. Prove that a collection of sets closed union and intersection is a lattice. 1 + 4

b) Prove that in a bounded distributive lattice (L, ∩ , ∪) an element cannot have more than one complement. 4

c) Find the sum of all four digits of even numbers that can be made with the digits 0, 1, 2, 3, 5, 6 and 8. 6

END