Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-III (All Branches) Examination December 2009

Subject code: 130001 **Subject Name: Mathematics III**

Date: 15 /12 /2009 **Time: 11.00 am – 2.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed **14**

(a) Solve: $\frac{dy}{dx} + y = x$.

(b) Evaluate the integral: $\int_0^\infty \exp(-x^2)dx$.

(c) Find $L\{\sin 2t \cos 2t\}$.

(d) State the generating function and integral representation for the Bessel function $J_n(x)$.

(e) Prove that: $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$.

(f) Show that: $x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$.

(g) Find the Fourier transform of the function

$$f(x) = \begin{cases} k, & 0 < x < a \\ 0, & otherwise \end{cases}$$

Q.2 **(a)** By using the method of Laplace transform solve the initial value problem: $y'' + 2y' + y = e^{-t}$, $y(0) = -1$ and $y'(0) = 1$. **07**

(b) Solve the following differential equations

(i) $2xy\,dx + x^2 dy = 0$ **02**

(ii) $\frac{dy}{dx} - y = e^{2x}$ **02**

(iii) $\frac{dy}{dx} + y = -\frac{x}{y}$ **03**

OR

(b) (i) Using the relationship between the beta and gamma functions, simplify the expression $B(m,n)\,B(m+n,p)\,B(m+n+p,q)$. **02**

(ii) Express $\int_0^1 x^m (1-x^n)^p\, dx$ in terms of Gamma function. **02**

(iii) State Legendre duplication formula. Hence prove that **03**

$$B(m,m)\ B\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \pi\ m^{-1}\ 2^{1-4m}.$$

Q.3 (a) Solve the initial value problem : **05**
$y'' + y' - 2y = 0$, $y(0) = 4$ and $y'(0) = -5$

(b) Given the functions e^{x} and e^{-x} on any interval [a, b]. **04**
Are these functions linearly independent or dependent?

(c) Using the method of variation of parameter solve the differential equation: $y'' + y = \sec x$. **05**

OR

Q.3 (a) Prove that: $\frac{d}{dx}[x^{n+1} \; J_{n+1}(x)] = x^{n+1} \; J_n(x)$. **05**

(b) Attempt (**any three**). **09**

(i) Express the polynomial $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials.

(ii) Show that $\int_{-1}^{1} P_m(x) P_n(x) dx = 0$, if $m \neq n$.

(iii) By using generating relation of Legendre polynomials, evaluate $P_n(-1)$.

(iv) Obtain the value of $\int_{-1}^{1} P_n^2(x) dx = 0$.

Q.4 (a) Find the Fourier series of the function $f(x) = x^2, -\pi < x < \pi$. **05**

(b) Obtain the Fourier series of periodic function **05**
$f(x) = 2x, -1 < x < 2, \; p = 2L = 2$.

(c) Obtain the Fourier transform of the function $\exp(-ax^2)$. **04**

OR

Q.4 (a) Using the method of undetermined coefficients, solve the differential equation: $y'' + 4y = 8x^2$. **05**

(b) Using the method of series solution, solve the differential equation: **04**
$y'' + y = 0$.

(c) Find the steady state oscillation of the mass-spring system governed by the equation: $y'' + 3y' + 2y = 20\cos 2t$. **05**

Q.5 (a) Attempt (**any two**) **04**

(i) Evaluate: $L^{-1}\left\{\frac{1}{(s+\sqrt{2})\,(s-\sqrt{3})}\right\}$.

(ii) Evaluate: $L^{-1}\left\{\frac{3}{s^2 + 6s + 18}\right\}$.

(iii) By using first shifting theorem, obtain the value of $L\{(t+1)^2 e^t\}$.

(b) Find the value of **04**

(i) $L\{t \sin \omega t\}$

(ii) $1 * 1$ where $*$ denote convolution product.

(c) (i) Evaluate: $L^{-1}\left\{\dfrac{s\,e^{-2s}}{s^2+\pi^2}\right\}$. **06**

(ii) Using convolution theorem, obtain the value of $L^{-1}\left\{\dfrac{1}{s(s^2+4)}\right\}$.

OR

Q.5 **(a)** Find the solution $u(x,y)$ of the partial differential equation $u_{xx}+v_{yy}=0$ by method of separation of variables. **07**

(b) Attempt (**any one**). **07**

(i) Prove that Laplacian *u* in polar coordinate is

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial u}{\partial x} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\ .$$

(ii) Find the potential inside a spherical capacitor consisting two metallic hemispheres of radius 1 ft separated by a small slit for reasons of insulation , if the upper hemisphere is kept at 110 volts and lower hemisphere is grounded.

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