S.E (Comp) Sem III (R)
Discrete Structure and Graph Theory

Con. 5653–09. (REVISED COURSE) SP–7370

Lib

(3 Hours) [Total Marks : 100

22/12/09
2-30 to 5-30

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from the remaining **six.**
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Prove by Mathematical Induction method – 5

$$1^2 + 2^2 + 3^2 + \dots\dots\dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

(b) If A = B = C = R where R is set of real number and $f : A \to B$, $g : B \to C$ are functions defined by $f(x) = x + 1$, $g(x) = x^2 + 2$, then find $(g \circ f)(x)$ and $(f \circ g)(2)$. 5

(c) Show that a group $(G, *)$ is abelian if and only if $(a * b)^2 = a^2 * b^2$. 5

(d) In a Boolean algebra, prove that $\overline{a \wedge b} = \overline{a} \vee \overline{b}$. 5

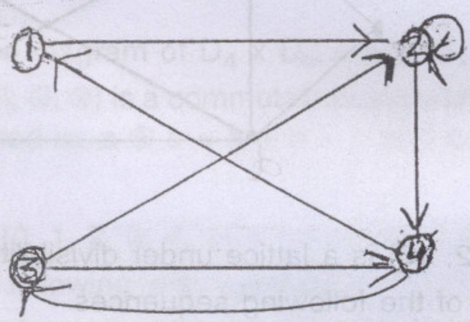
2. (a) Show that the relation $R = \{(x, y) \mid x - y$ is divisible by 4; where x, y are int egers$\}$ is an equivalence relation. Write the equivalence classes given by R. 6

(b) Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with initial conditions $a_0 = 1$, $a_1 = -1$. 6

(c) Explain Quantifiers. Negate the statement '$\sqrt{2}$ is not a rational number'. 4

(d) Draw all Hasse diagrams of posets with three elements. 4

3. (a) Find the transitive closure of the relation R on set A defined by the given digraph using Warshall's Algorithm 6

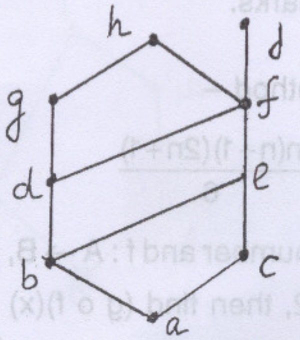


(b) Show that the (2, 5) encoding function $e : B^2 \to B^5$ defined by 6

e(00) = 00000
e(01) = 01110
e(10) = 10101
e(11) = 11011

is a group code. Find the minimum distance.

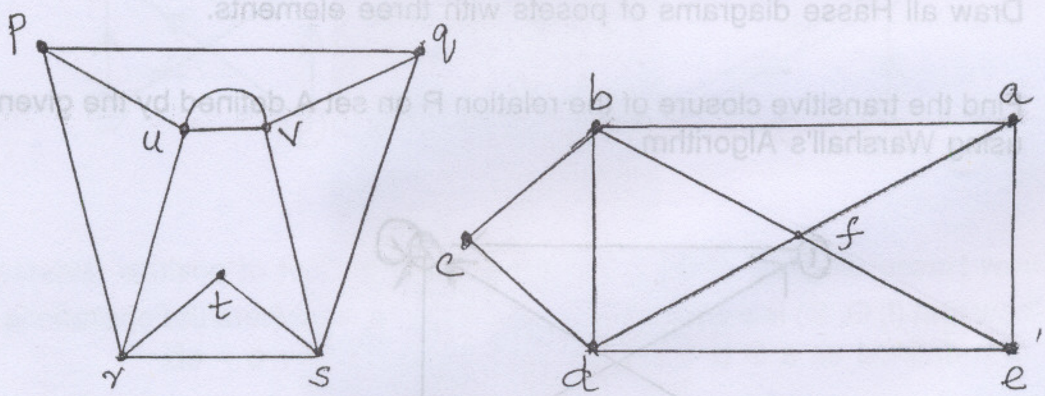
(c) Find the lower and upper bounds of the subsets {a, b, c} and {a, c, d, f} of given poset **4**



(d) Show that if any five integers from 1 to 8 are selected, then the sum of at least two of them will be 9. **4**

4. (a) Consider the relation R on set of integers defined as xRy iff $y = x^k$; k is positive integer. Show that R is a partial order relation. **6**

(b) Determine the Eulerian path and Hamiltonian path, if exist, in the following graph. **6**



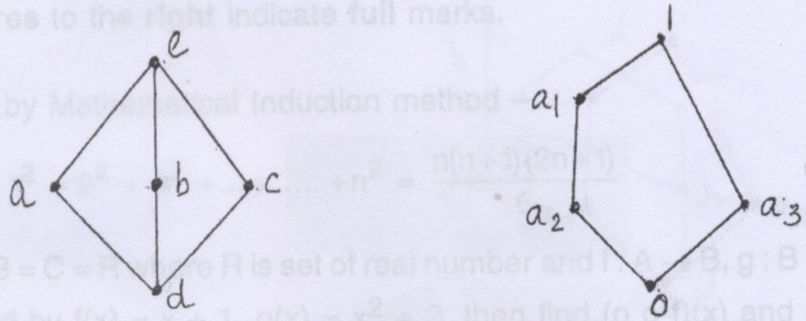
(c) Check if the set A = {2, 4, 12, 16} is a lattice under divisibility. **4**

(d) Find the generating function of the following sequences **4**

(i) 1, 0, –1, 0, 1, 0 –1, 0, ... (ii) 1, 1, 1, 1, 1,.............

5. (a) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Decode the following words relative to maximum likelihood decoding function (i) 011001 (ii) 101011 (iii) 111010. **6**

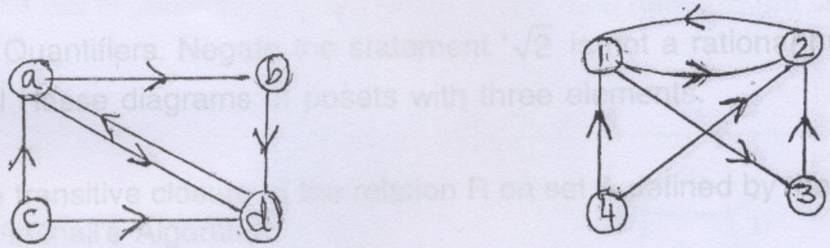
(b) Show that the lattices given in the following Hasse diagrams are non distributive 6



(c) Find the number of vertices of the graph having 16 edges if degree of each vertex is 2. 4

(d) For sets A, B, C prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ 4

6. (a) Define isomorphic graphs. Determine whether the given graphs are isomorphic or not 6



(b) Draw Hasse diagram of $D_4 \times D_9$ where D_n is the set of positive divisors of n. 6

(c) Show that $(I, \oplus, \otimes)$ is a commutative ring with identity where the operations $\oplus$ and $\otimes$ are defined as $a \oplus b = a + b - 1$ and $a \otimes b = a + b - ab$. 8

7. (a) Show that {0, 1, 2, 3, 4, 5} is an abelian group under the operation+6. 6

(b) Define the following with example 6

(i) Ring homomorphism

(ii) Field

(iii) Spanning tree.

(c) Show that the function $f : R - \{2\} \rightarrow R - \{0\}$ where R is set of real numbers defined by $f(x) = \dfrac{1}{x-2}$ is a bijection. Find its inverse. 8
