B4.2-R3: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) i) Let A={0, 1, 2, 3, …}. Define function *f, g* and *h* from A to A by *f(x)*=2*x, g(x)*=*x*+1 and

$$h(x) = \begin{cases} 0, & \text{if } x \text{ is odd} \\ 1, & \text{if } x \text{ is even} \end{cases}$$

where $x \in A$. (Treat '0' as an even number). Find (f∘g)∘h and h∘f.

ii) If $X = \{ n^2 : n \text{ is a positive integer} \}$ and $Y = \{ n^3 : n \text{ is a positive integer} \}$, than find X ∩ Y.

b) Draw the Hasse diagram of D_8, the lattice factors of 8 under the relation of 'divisibility'. Is it totally ordered?

c) Let *f* and *g* be permutations defined on the set {1,2,3,4,5} by $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 5 & 1 & 4 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$. Find f^{-1} X and f •g. Also, express *g* as a product of transpositions.

d) A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways they can be seated if
  i) all the sisters sit together.
  ii) no two sisters sit together.

e) Define the terms 'Monoid' and 'Graph'. Give an example of a group which is not a monoid.

f) Define the term 'Graph'? In any graph G=(V, E), show that the number of vertices of odd degree is always even.

g) Define 'Phrase Structure Grammar' and a 'Finite State Machine'. Let A={a,b,c}. Let L_1 and L_2 be the languages defined on A by L_1={a,ab,ac,abc} and L_2={aba,aabaa}. Find L_1L_2 and L_2^{-2}.

(7x4)

2.

a) Let A={a,b,c}. Let R be a relation defined on A by R={(a,b), (b,a),(c,c)}. Find the reflexive, symmetric and transitive closures of the relation R.

b) i) Define 'difference' and 'symmetric difference' of two sets.
  ii) Find the number of integers between 1 and 250 that are divisible by either 6 or 8.

c) i) State the 'law of detachment' and 'Chain rule'.
  ii) Using the laws of logic, prove that $\left(\overline{p \wedge q}\right) \rightarrow \left(\bar{p} \vee \left(\bar{p} \vee q\right)\right) \Leftrightarrow \left(\bar{p} \vee q\right)$.

(6+6+6)

3.

a) Define 'Sublattice of a lattice'. Let A={a,b,c}. Let L be the lattice of power set of A under the relation of 'Contained in'. Let S={Φ,{a}, {b,c}, A}. Then

i) find {a,b} Λ {b,c} in A.

ii) Is the set S, a sublattice of L? Justify. Also, draw the Hasse diagram of S.

b) Define the term 'Boolean Algebra'. State DeMorgan's laws in a Boolean algebra. Prove any one of them

c) Using the principle of mathematical induction, prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for all n=2,3,4,5, ….

(7+5+6)

4.

a) Define the term 'Cyclic group'. Let G={1,2,3,4}. Show that G is a group under the binary compositon ⊙ 5, the multiplication modulo 5. Is G cyclic? Justify.

b) i) Let G be any group in which every element is its own inverse. Show that G is abelian.

ii) Let (a,∗) be a commutative semi-group, show that if a∗a = a and b∗b = b, then (a∗b)∗(a∗b) = a∗b.

c) Solve the simultaneous congruences $3x \equiv 1 (mod\ 5), 2x \equiv 3 (mod\ 7)$.

(6+6+6)

5.

a) Solve the recurrence relation: $a_n - 7a_{n-1} + 10a_{n-2} = 3^n$ given that $a_0=0$ and $a_1=1$.

b) In any Boolean algebra define terms 'PDNF' and 'PCNF'. Express the following Boolean function and its complement in PDNF:

(x,y,z)	*f(x,y,z)*
(0,0,0)	1
(0,0,1)	0
(0,1,0)	1
(0,1,1)	0
(1,0,0)	0
(1,0,1)	1
(1,1,0)	0
(1,1,1)	1

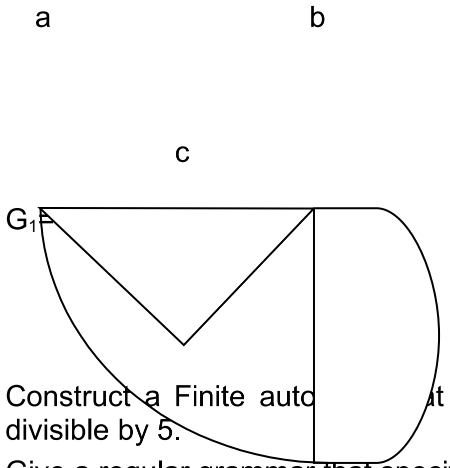
c) Simplify, using Karnaugh map the following Boolean function $f(x,y) = \sum (1,2,3,5)$.

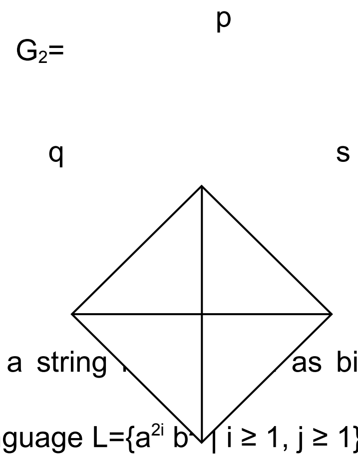
(6+7+5)

6.

a) i) Define the terms 'bipratite graph' and 'spanning tree'. Give an example of a graph which is Enlerian but not Hamiltorian.

ii) Are the following graphs isomorphic? Justify.





b) i) Construct a Finite auto … at accepts a string … as binary number and divisible by 5.

ii) Give a regular grammar that specify the language $L=\{a^{2i} b^{j} \mid i \geq 1, j \geq 1\}$.

c) Let M be a FSM given by following table. Construct an equivalent FSM corresponding to the machine M.

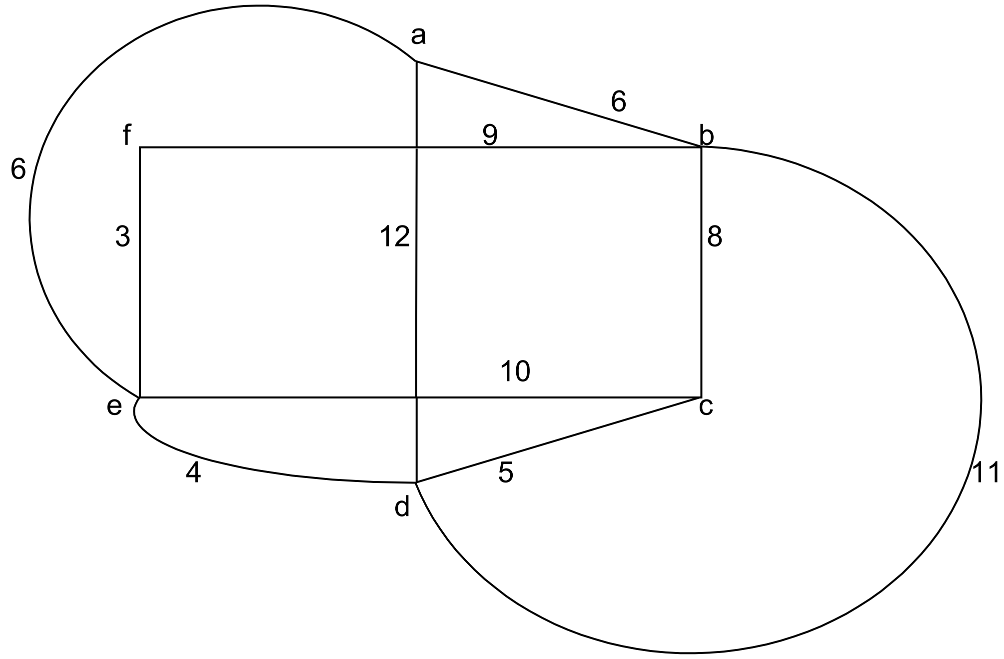
State	Input		Output
	1	2	
A	B	C	0
B	F	D	0
C	G	E	0
D	H	B	0
E	B	F	1
F	D	H	0
G	E	B	0
H	B	C	1

(6+7+5)

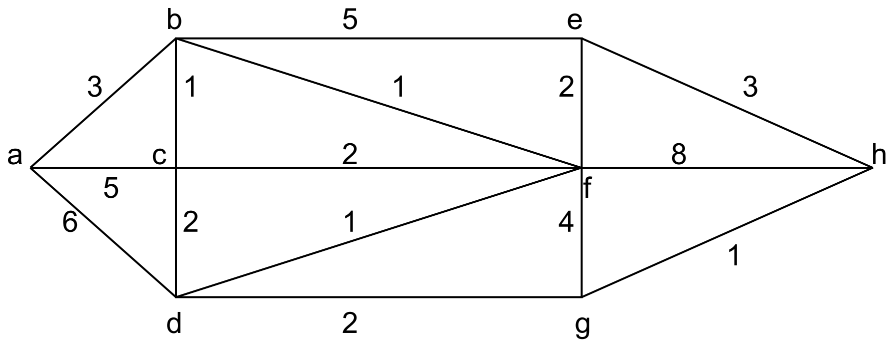
7.

a) In a cricket tournament, seven teams participate. Each team play with the remaining six teams and each team win at least one game. Find the total number of matches to be played. Also, using pigem-hole principle, show that there are at least two teams having the same number of wins.

b) Determine a minimum spanning tree for the graph shown in below:



c) Apply DijKstra's algorithm to determine a shortest path from the vertex 'a' to the vertex 'h' in the following graph:



(5+6+7)