B4.2-R3: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) i) If A= ϕ and B={{{a}}, ϕ } then Find (P(P(A))) and P(B). (P denotes Poset Set).

ii) Let A={1, 2, 3, 4, 5} and B={4, 5, 6, 7}. Compute i) A-B, ii) AXB

b) Show that every cyclic group of order n is isomorphic to the group $<Z_n, +_n>$.

c) Let X={2, 3, 6, 12, 24, 36} and the relation ≤ be such that x ≤ y if x divides y. Draw the Hasse diagram of $< X, \leq >$.

d) What do you mean by DUAL of a statement in Boolean algebra?

e) Let p: Food is good

q: service is good

r: restaurant is 5-star

Write following in Symbolic notations

i) It is not true that five star restaurant always means for good food and good service.

ii) Food is good but service is poor.

f) How many ways can the letters in word MISSISSIPI can be arranged?

g) Let A= {a, b}. Write regular expressions such that L (r) which consists of all words w, where fifth character from right end is always a.

(7x4)

2.

a) Find the Principal Conjunctive Normal Form of the formula $(\sim p \rightarrow R) \wedge (Q \leftrightarrow P)$.

b) Test the validity of the following argument:

If I study, then I will not fail in Mathematics

If I do not play basket ball, then I will study

But I failed Mathematics

Therefore, I must have played basketball.

c) In a room containing 28 females, there are 18 females who speak English, 15 females speak French and 22 speak German. 9 females speak both English and French, 11 females speak both French and German whereas 13 speak both German and English. How many females speak all the three languages?

d) Find all Partitions of S={1, 2, 3}.

(5+5+5+3)

3.

a) Prove that both join and meet operations in Lattice algebra are associative.

b) Prove that, in a distributive lattice, if an element has a complement then this Complement is Unique.

c) Find a minimal sum for the Boolean expressions:

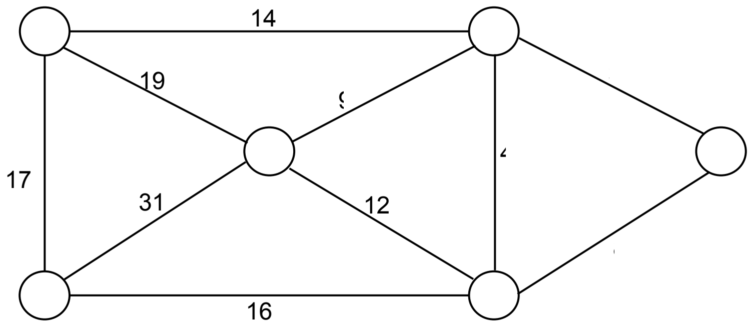
E= x + x'yz + xy'z

d) Prove that the relation, “Divides” is a POSET.

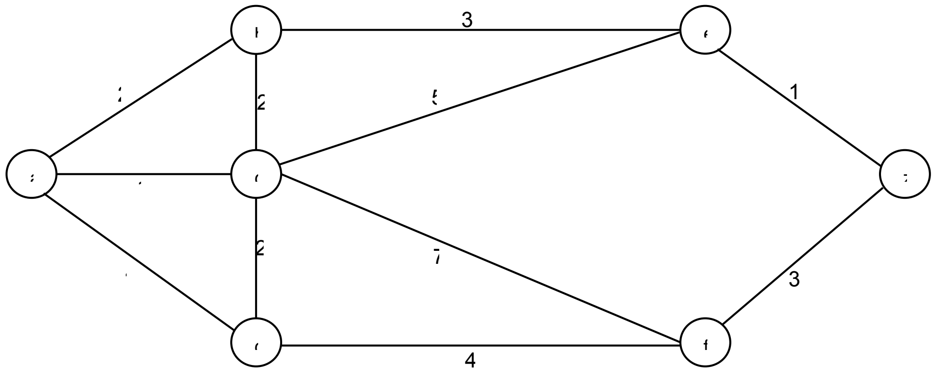
(5+5+5+3)

4.

a) For the following graph find its spanning tree of minimal cost using kruskal algorithm.



b) Apply the Dijkistra's algorithm to find the shortest path from a to z from given figure.



c) Define briefly the followings
   i) Cut Set, ii) Eulerian Circuit, iii) Bipartite graph, iv) Hamiltonian Path

(6+8+4)

5.

a) Let A= {0, 1} construct a deterministic finite automation M such that L(M) will consist of
   i) Those words in which number of 0's and 1's is even
   ii) Those words in which number of 1's are odd

b) Show that the language L= $\{a^m b^m$: m positive$\}$ is not regular.

c) Define a Phase structure grammar.

(10+6+2)

6.

a) Find integers m and n such that 512 m + 320 n = 64.

b) Prove by mathematical inductions the following, $2^n > n^3$ for $n \geq 10$.

c) If Fn satisfied the Fibonacci relation for the Fibonacci series (1, 1, 2, 3,…) define by the recurrence relation, $F_n = F_{n-1} + F_{n-2}$, F0 = F1, then find a formula to find nth Fibonacci number.

(5+6+7)

7.

a) Prove that composition of two homomorphism is also a homomorphism.

b) If {G, *} is an abelian group, show that $(a * b)^n = a^n * b^n$, for all a, b Є G, where n is a positive integer.

c) A bag can contain six white balls and five red balls. Find the number of ways, four balls can be drawn from the bag if two must be white and two red.

d) State pegions holes principle.

(5+5+5+3)