B4.2-R3: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let $D_n = (0, 1/n)$ where $n \in N$: the set of positive integers. Find $\cap_{i \in N} D_i$.

b) Find the power set of A = {(a,b),c}.

c) Let V_N = {S}, V_T = {a}, P = {S $\rightarrow$ SS} with S as a starting symbol, find L(G) where G = (V_N, V_T, P, S).

d) How many vertices do the following graphs have if it contains 21 edges, 3 vertices of degree 4 and others each of degree 3?

e) What is the coefficient of $x^3 y^2 z^2$ in $(x + y+ z)^9$.

f) A function f is defined on the set of integers as follows:

$$F(x) = \begin{matrix} x & \text{if } 0 \le x < 1 \\ x+2 & \text{if } 1 \le x < 3 \\ 4x-5 & \text{if } 3 \le x > 5 \end{matrix}$$

Find its domain, range and check for one-one or many one .

g) Simplify f(x,y) = x y ' + xy + x'y

(7x4)

2.

a) If R be a relation on the set of integers Z defined by

R = { (x y) : $x \in Z$, $y \in Z$: x – y is divisible by 3}

Describe the distinct equivalence classes of R.

b) Show that the mapping f : R$\rightarrow$ R be defined by f(x) = ax +b where a, b, x $\in$ R, a $\neq$ 0 is invertible. Define its inverse.

c) Consider the poset A = ({1,2,3,4,6,9,12,18,36}, /), find the greatest lower bound and the least bound of the sets {6, 18} and {4, 6, 9}.

(6+6+6)

3.

a) Is the proposition s a valid conclusion from the premises p $\Rightarrow$ q, p $\Rightarrow$ r, $\neg$ (q r) and s $\vee$ p?

b) Use Karnaugh map to simplify the expression

X = A'B'CD + A'B'CD' + AB'CD' + AB' CD'

c) Use mathematical induction to prove that 1+2+3+…+N= $\frac{N(N+1)}{2}$.

(6+6+6)

4.

a) Let ({a, b}, *) be a semigroup where a*a=b. Show that:

 i) a*b = b*a

 ii) b*b = b

b) Solve the recurrence relation

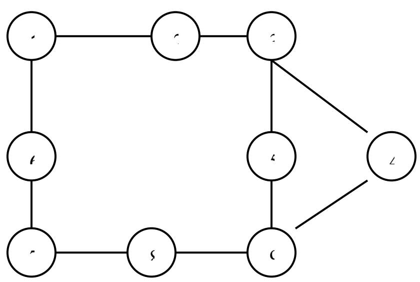
$$a_n + 5a_{n-1} + 6a_{n-2} = 3n^2.$$

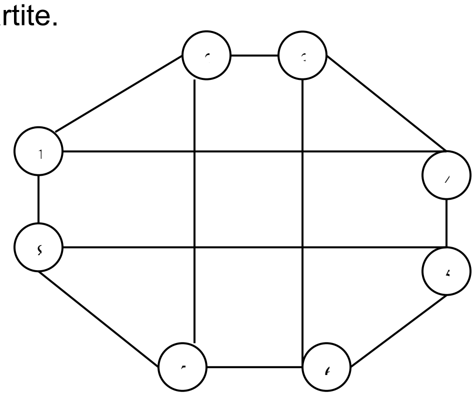
c) A computer password consists of a letter of the alphabet followed by three or four digits. Find the total number of passwords that can be formed, and also the number of passwords in which no digit repeats.

(6+6+6)

5.

a) Determine whether or not each of the graphs is bipartite. In each case give the bipartition sets or explain why the graph is not bipartite.





b) Define: i) Hamiltonian Circuit

 ii) Enlerian Path

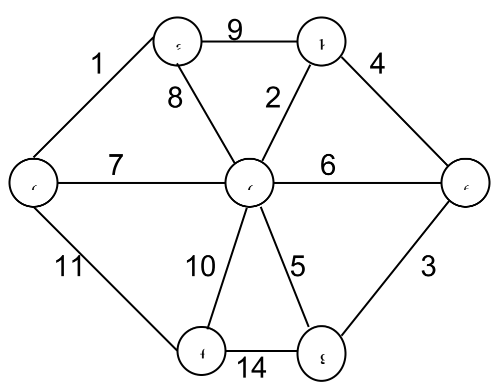
 iii) Planar Graph

c) Show that the graph K_5 is not planar.

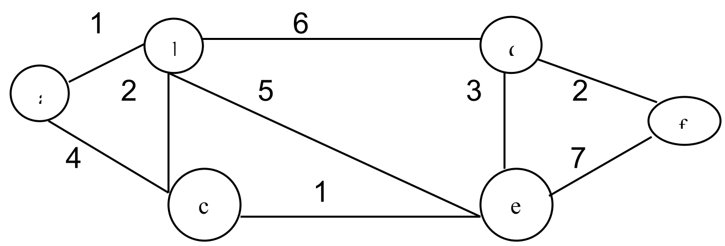
(6+6+6)

6.

a) Show how Kruskal's algorithm finds a minimum spanning tree of the following graph:



b) Apply Dijkstra's algorithm to find the shortest path from a to f. Workout all the intermediate steps as per the algorithm.



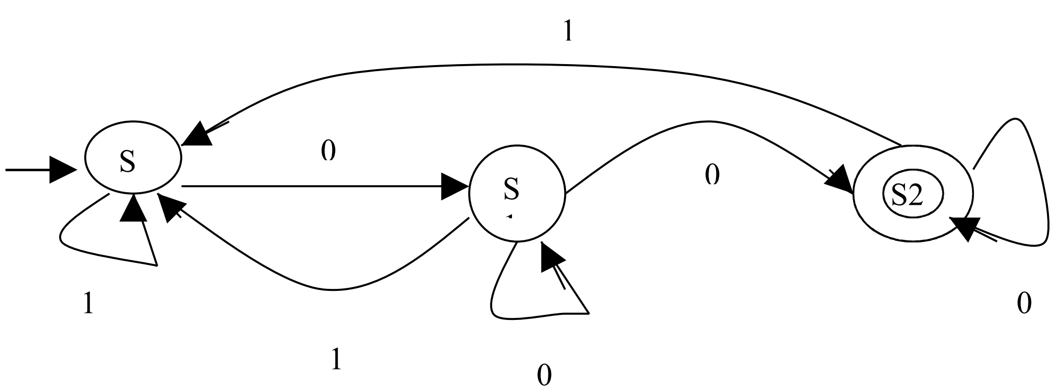
c) Show that the greatest common divisor is unique.

(7+7+4)

7.

a) Let $V_N = \{S, B\}$, $V_T = \{a, b\}$, $P = \{S \rightarrow a\,B\,a,\ B \rightarrow a\,Ba,\ B \rightarrow b\}$. Find L(G).

b) Consider the finite state automaton A shown below. What is the language accepted by A.



c) State pumping lemma for regular language.

(7+7+4)