Third Year B.Sc. Degree Examination
Aug/Sept 2009
Directorate of Distance Education Course

MATHEMATICS (PAPER - IV)

Time : 3 Hours | Max. Marks : 90

Note : Answer any SIX of the following.

PART - A

1. a) i) Evaluate $\int_c xy\,dx + x^2z\,dy + xyz\,dz$ where c is the curve $x=e^t$, $y=e^{-t}$, $z=t^2$, $0 \le t \le 1$ — 2

ii) Evaluate $\int_0^1 \int_0^2 (x+y)\,dx\,dy$ — 2

b) If c is the curve leading from (–1,2,3) to (3,2,–1)

Evaluate $\int_c (3x^2-3yz+2xz)\,dx + (3y^2-3xz+z^2)\,dy + (3z^2-3xy+x^2+2yz)dz.$ — 5

c) Evaluate $\iint_d \frac{x^2y^2}{x^2+y^2}\,dx\,dy$, where D is the annular region between the circles $x^2+y^2=2$ and $x^2+y^2=1$. — 6

2. a) i) Evaluate $\int_1^2 \int_0^x (x+2y)\,dy\,dx$ — 2

ii) Evaluate $\int_0^1 \int_1^2 \int_1^2 x^2yz\,dz\,dy\,dx$ — 2

b) Derive the formula for the surface area of the sphere of radius 'a' units. — 5

c) Find the volume common to the cylinder $x^2+y^2=a^2$ and $x^2+z^2=a^2$. — 6

3. a) i) Prove that $\Gamma(n+1) = n\,\Gamma(n)$. — 2

ii) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\,d\theta$. — 2

b) Prove that $\int_0^{\infty} \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx = 2\,\beta(m,n)$ — 5

c) Prove that $\int_0^a y^4 \sqrt{a^2-y^2}\,dy = \frac{\pi a^6}{32}$ — 6

4. a) i) Show that a constant function f(x)=k is R-integrable and $\int_a^b k\,dx = k(b-a)$. **2**

ii) Prove that lower Riemann integral can never exceed the upper Riemann integral. **2**

b) State and prove Darboux theorem. **5**

c) Show that (3x+1) is integrable on [1,2] and $\int_1^2 (3x+1)\,dx = 11/2$. **6**

PART - B

5. a) i) Find the part of the complementary function of

$$(x+1)\frac{d^2y}{dx^2} - 2(x+3)\frac{dy}{dx} + (x+5)y = e^x \text{ where } x \neq -1.$$ **2**

ii) Verify the condition for exactness of the equation

$$(2x^2+3x)\frac{d^2y}{dx^2} + (6x+3)\frac{dy}{dx} + 2y = (x+1)e^x.$$ **2**

b) Solve $\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x$ by changing the independent variable. **5**

c) Solve $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$, x>0 by changing the dependent variable, given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$ when x=1. **6**

6. a) i) Find the Wronskian of $\frac{d^2y}{dx^2} + 9y = \sec 3x$. **2**

ii) Write the complementary functions for the cases $1-P+Q=0$ and $P+Qx=0$. **2**

b) Solve $x^2y'' + xy' - y = 2x^2$, $x > 0$ given that $\frac{1}{x}$ is a part of the complementary function. **5**

c) Solve $y'' - y = \frac{2}{1+e^x}$ by the method of variation of parameters. **6**

7. a) i) Verify the condition of integrability of the function

$$(2x^2-z)z\,dx + 2x^2yz\,dy + x(z+x)\,dz = 0$$ **2**

ii) Form a partial differential equation by eliminating arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. **2**

b) Solve $\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$ **5**

c) Solve $(3z-4y)p + (4x-2z)q = (2y-3x)$ **6**

8. a) i) Write the formula for Fourier constants a_n & b_n. 2

ii) If $f(x) = \begin{cases} -K & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$ find "a_0"

given by $f(x+2\pi)=f(x)$. 2

b) Obtain the Fourier series for the function defined by

$$f(x) = \begin{cases} -1 & -3 < x < 0 \\ 0 & x=0 \\ 1 & 0 < x < 3 \end{cases}$$ 5

c) Find the half range cosine series for

$f(x) = \begin{cases} \pi/3 & 0 \le x \le \pi/3 \\ 0 & \pi/3 \le x \le 2\pi/3 \\ -\pi/3 & 2\pi/3 \le x \le \pi \end{cases}$ then show that

$$f(x) = \frac{2}{\sqrt{3}} [\cos x - \frac{1}{5} \cos 5x + \frac{1}{7} \cos 7x + \text{..........}]$$ 6

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