BE4-R3: PRINCIPLES OF MODELLING AND SIMULATION

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Suppose you are developing the simulation model of a small petrol pump on a highway where the incoming cars are being served by a single service person. Assume the arrival of the cars as well as their service times are random. Identify some states, events and entities for this simulation model.

b) Name some important software packages for discrete event simulation. Discuss briefly how would you select suitable software for discrete event simulation.

c) Compute the expected value and variance of a set of pseudo-random numbers, which are uniformly distributed over (0, 1).

d) What are some characteristics of the service mechanism in a queuing system? Why are they important?

e) Describe algorithm for generating random variate with the distribution function

$$F(x) = P(X \le x) = \begin{cases} 1 - e^{-x/\beta} & \text{if } x \ge 0 \\ 0 & \text{otherwise} \end{cases},$$

f) Contrast between deterministic and stochastic systems. Give an example of a real life system which requires development of a stochastic model for its description.

g) Define the terms “factor” and “response” in the context of a simulation experiment. Give an example.

(7x4)

2.

a) A random variable X has the following empirical distribution:

x :	1	2	4	6	8	10
$f(x)$:	0.10	0.20	0.25	0.20	0.15	0.10

Plot the cumulative distribution and find the value of X corresponding to the following two digit random numbers, 0.05, 0.45, 0.62, 0.93.

b) What are the different tests for randomness in pseudorandom numbers? Why are these tests required?

c) Generate a sequence of ten two digit random numbers using the mixed congruential generator:

$$r_{n+1} = (21 r_n + 53) \bmod 100, \quad r_0 = 52$$

(6+8+4)

3.

a) What do you mean by discrete event systems? Discuss some considerations for building a discrete event simulation model?

b) "In a fast-food restaurant, customers arrive randomly. Some of the customers go to the sandwich counter, while the rest go to the soup counter. Queues are usually formed in front of both the counters having exponentially distributed service times. After the customers eat, all of them pay at a single cash counter, which again has an exponentially distributed service time. Queues are formed there as well. A customer leaves after the payment is made".

How would you develop a simulation model for the fast-food restaurant? How will you collect data for validation of the model?

(6+12)

4.

a) There is a train to the main city every 25 minutes at a suburban station from 7.20 AM to 9.00 AM. A certain commuter does not know the train timetable, but arrives randomly (uniformly distributed) between 8.00 AM and 8.30 AM every morning. Find the probability that the passenger does not have to wait for more than 7 minutes for a train?

b) A businessman compiled data related to his daily receipts and payments for a year based on the data, he assigned the following probabilities:

Daily Receipt (in Rs.)	Probability		Daily Payments (in Rs.)	Probability
3000	0.20		4000	0.30
5000	0.30		6000	0.40
7000	0.40		8000	0.20
12000	0.10		10000	0.10

Given random numbers for daily receipts: 03, 91, 38, 55, 17, 46, 32, 43, 69, 72,
and random numbers for daily payments: 61, 96, 30, 32, 03, 88, 48, 28, 88, 18.

simulate the daily pattern of receipts and payments for the next 10-day period. Assuming further that the beginning balance is Rs. 8000, what is the estimated balance at the end of the 10-day period? What is the highest daily balance during the 10-day period? What is the average daily balance?

(6+12)

5.

a) In a barber shop with a single barber, there are two chairs for waiting customers. On an average, one customer arrives every 10 minutes and each customer takes 5 minutes for getting served. Making suitable assumptions, find:

i) the probability that an arriving customer will find at least one chair free,
ii) the probability that an arriving customer will have to stand,
iii) expected waiting time of a customer in the queue.

b) What do you mean by balking, reneging, and jockeying in a queuing problem? Give real-life examples, where you encounter such phenomena.

c) The life of a mechanical instrument is exponentially distributed with a failure rate of one failure in every 4000 hours on the average. Find out the probability that the instrument will last longer than its mean life of 4000 hours. Also find the probability that the instrument will last between 3000 and 4000 hours.

(6+6+6)

6.

a) Describe the classification of mathematical models. Also bring out the limitations of simulation studies.
b) Outline some strategies for getting a valid simulation model? Enumerate them.
c) Discuss the needs for good documentation and software support in the validation of a simulation model.

(6+6+6)

7.

a) Why is it necessary to carryout output analysis of simulation models? Are the output data from simulation normally distributed? Discuss your answer with an example.
b) Discuss variance-reduction techniques for carrying out simulation experiments.
c) How would you estimate the length of the simulation run in discrete event simulation?

(6+6+6)