BE4-R3: PRINCIPLES OF MODELLING AND SIMULATION

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A telephone exchange receives 100 calls a minute on average according to a Poisson process. What is the probability that no calls are received in an interval of five seconds?

b) Give an example each of the following systems:
   i) a Deterministic system
   ii) a Stochastic system
   iii) a Continuous system
   iv) a Discrete system

c) What is the difference between a simulator and a simulation language?

d) What is GPSS? Describe its features.

e) Name four principal entities, attributes and activities to be considered if you were to simulate the operation of a petrol filling station.

f) Consider the following relation used for generating random numbers

$n_{i+1} = (a_{ni} + b) \bmod m$ with $n_0, a, b, m \in N;$

$i = 0, 1, 2, \ldots, m - 1$

Given $a = 17, b = 0, m = 100, n_0 = 100$, find n_1, n_2, n_3, n_4.

g) Using random numbers uniformly distributed over (0, 1), outline a procedure based on Monte Carlo method to estimate $\prod$.

(7x4)

2.

a) Traffic to a message switching centre arrives according to a Poisson process at an average rate of 240 messages per minute. The line has a transmission rate of 800 characters per second. The message length distribution including control characters is approximately exponential with an average length of 176 characters: -
   i) Compute mean arrival and mean service rates.
   ii) Calculate the server utilisation.
   iii) Find the mean number of messages in the system.
   iv) Determine the average response time.

b) Why is an analytical model preferred over a simulation model? Discuss the advantages and disadvantages of simulation.

([2+2+3+3]+[3+5])

3.

a) What is antithetic sampling? Bring out its significance in the context of simulation experiments.

b) Explain, through a real life example, the significance of list processing in simulation languages.

c) Compare any two simulation languages with regard to their capability to aid the modeller in debugging.

(6+6+6)

4.

a) A certain production process produces on the average seven percent defective items. Defective items occur randomly. Items are packaged for sale in lots of five. The production manager wants to know what percentage of the lots contained no defective item. Solve this problem using Monte Carlo simulation. You may as well compare your simulated results with those obtainable using the analytical methods. If there remains any difference between such results, then account for the same.

b) Discuss about the output analysis for steady state simulation. Also mention the consideration that a simulation analyst has to choose for simulation run length.

(10+8)

5.

a) Compare validation in simulation to the validation of theories in physical sciences.

b) Distinguish between transient simulations and steady state simulations in the context of analysis of simulation output data.

(9+9)

6.

a) Let G(t) be the amount of glucose present in the blood stream of a patient at time t. Assuming that the glucose is injected into the blood stream at a constant rate of C gm per minute, At the same time, it is converted and removed from the blood stream at a rate proportional to the amount of glucose present.

i) Set up a differential equation for G(t).

ii) If $G(0) = G_0$, what is the equilibrium level of glucose in the blood stream?

b) A company establishes a pension fund for the employees by setting Rs. P a month for each employee. The accumulated fund is invested and earns 5% per annum. The work force is expected to grow at 3% per annum. The company wants to study the soundness of its plan by simulating the effects of different assumptions about average length of service and average length of retirement.

i) Draw a System Dynamics flow chart for the simulation.

ii) Formulate the mathematical model.

(10+8)

7.

a) An algorithm for generating X is as follows:

i) Generate U ~ U (0, 1)

ii) Let V = F(a) + [F(b) – F(a)] U

iii) Return $X = F^{-1}(V)$

Show that X defined by this algorithm has the distribution function $F^*(x)$ defined by

$$F^*(x) = \begin{cases} 0 & ,\text{if } x < a \\ \dfrac{F(x) - F(a)}{F(b) - F(a)} & \text{if } a \le x \le b \end{cases}.$$

b) Describe the tests to check randomness in pseudo-random numbers.

(10+8)