050(E)

(MARCH, 2009)

Time : 3.00 Hours]   **[Maximum Marks : 100**

Instructions :

1. **All** the questions are **compulsory.**

2. Write your answers according to the instructions given below with the questions.

3. Begin each section from a *new* page.

SECTION - A

Questions from 1 to 15 are multiple choice questions, each carrying ONE mark. Write the letter of the correct option (A), (B), (C) or (D) in your answer book from the alternatives.   **15**

1. Measure of the angle between lines $x = 1$ and $\sqrt{3}x + y - 4 = 0$ is .....

   (A) $2\pi/3$ (B) $\pi/6$

   (C) $7\pi/6$ (D) $\pi/2$

2. If the lines $5x - ky - 7 = 0$ and $2x + 3y + 5 = 0$ are mutually perpendicular to each other, then $k = $ ....

   (A) $-\dfrac{10}{3}$ (B) $\dfrac{10}{3}$

   (C) $-\dfrac{15}{2}$ (D) $\dfrac{15}{2}$

3. The equation of the line passing through the diametric points of the circle $(x - 2)^2 + (y - 3)^2 = 25$ is ....
   (A) $2x + y = 5$ (B) $(x - 2) + (y - 3) = 25$
   (C) $x + y = 5$ (D) $x + 2y = 10$

4. Parametric equation of $y^2 = 12x$ are .... ($t \in$ R).
   (A) $(6t, 3t^2)$ (B) $(4t^2, 3t)$
   (C) $(4t^2, 8t)$ (D) $(3t^2, 6t)$

5. Measure of the angle between asymptotes of $x^2 - y^2 = 1$ is ....

   (A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{3}$

   (C) $\dfrac{\pi}{2}$ (D) 0

6. If $\bar{x} = (2, 0)$, $\bar{y} = (3, 0)$, then $|\bar{x} + \bar{y}|$ .... $|\bar{x}| + |\bar{y}|$.
   (A) $=$ (B) $<$
   (C) $>$ (D) None of them.

7. Projection of $\bar{i}$ in the direction of $\bar{j}$ is ....
   (A) $\bar{o}$ (B) $\bar{i}$
   (C) $\bar{j}$ (D) $\bar{k}$

8. If the directions of $\dfrac{x-1}{c} = \dfrac{y+2}{-2} = \dfrac{z-3}{4}$ and $\dfrac{x-5}{1} = \dfrac{y-3}{1} = \dfrac{z+1}{c}$ are same, then $c = $ ....
   (A) -2 (B) 2
   (C) 4 (D) -4

9. The radius of the Sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 1 = 0$ is .....
   (A) 4 (B) $\sqrt{2}$
   (C) 2 (D) $\sqrt{13}$

10. $\displaystyle\lim_{x \to \infty} x\left(\sqrt[x]{2} - 1\right) = $ ....

    (A) $\log_2 x$ (B) $\log_2 e$
    (C) $\log_e 2$ (D) $\log_e x$

11. If $x = 3t^2 - 6t + 5$ and $v = 0$, then $t = $ ....
    (A) 0 (B) 1
    (C) 5 (D) -1

12. The rate of increasing volume of the Sphere w.r.t., its surface area is ....
    (A) $2r$ (B) $r/2$
    (C) $r/4$ (D) $r/3$

13. $\int \left(\sin \dfrac{x}{2} + \cos \dfrac{x}{2} \right)^2 dx = \ldots\ldots\ldots + c.$

(A) $(x - \cos x)$ (B) $(x + \sin x)$

(C) $(x + \cos x)$ (D) $(x - \sin x)$

14. The area of the region bounded by $y = \tan x$, X axis, $x = 0$ and $x = \pi/4$ is .... units.

(A) $2 \log 2$ (B) $\dfrac{1}{2} \log 2$

(C) $\log 2$ (D) 1

15. The order and degree of the differential equation $\sqrt[3]{\left(\dfrac{d^2 y}{dx^2} \right)^2} = \sqrt{1 + \left(\dfrac{dy}{dx} \right)^2}$

are ....
(A) $4, 4$ (B) $2, 4$
(C) $4, 2$ (D) $2, 2$

SECTION - B

Answer the following questions from S.Nos. 16 to 30.
Each question carries ONE mark.   **15**

16. Find the point $P(a, b)$ on the line, whose parametric equations are $x = 2t + 1$ and $y = 1 - t$, $t \in R$, so that $a + b = 1$.

17. Find the Cartesian equation of the Circle whose parametric equations are $x = -1 + 2 \sin \theta$ and $y = 1 + 2 \cos \theta$, $\theta \in (-\pi, \pi)$.

18. For the Parabola $y^2 = -8x$, obtain length of the Latus rectum and the end points of latus rectum.

19. Find the measure of eccentric angle for Ellipse

$\dfrac{x^2}{100} + \dfrac{y^2}{25} = 1$ at $(-6, 4)$ which belongs to it.

OR

Find the equation of the Ellipse, whose vertex is $(\pm 5, 0)$ and foci $(\pm 4, 0)$.

20. Find direction angles of $\bar{x} = (3, 0, -4)$.

21. Decide the direction for vectors $\bar{x} = (1, 3, 1)$, $\bar{y} = (2, 6, -2)$ whether same, opposite or different.

22. Find the displacement of a particle $3\bar{i} + 2\bar{j} - 5\bar{k}$ due to the force $2\bar{i} - \bar{j} - \bar{k}$ and find the work done.

23. Find the perpendicular distance from P(1, 2, 3) to line
$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}.$$

24. Obtain the equation of the Plane passing through
(3, 4, 2), (2, 2, –1) and (7, 0, 6).

25. Obtain $\dfrac{d}{dx}(x^x)$.

26. Find : $\dfrac{d}{dx}\left(\sin^{-1}\dfrac{x}{a}\right)$ where $a \neq 0$.

27. Find : $\displaystyle\int_{-1}^{1} \sin^3 x \, \cos^4 x \, dx$.

OR

Find the value of $\displaystyle\int_{0}^{\sqrt{2}} \sqrt{2 - x^2} \, dx$.

28. Obtain $\displaystyle\int \left[e^{a \log x} + e^{x \log a}\right] dx$.

29. Find the area of the region bounded by curve $x = y$, X- axis and the lines $x = 2$ and $x = 3$.

30. Find the differential equation for the family of the lines $y = mx + c$, where m, c are arbitrary constant.

SECTION - C

Answer the following questions from Nos. 31 to 40.

Each question carries TWO marks. Answer as directed in the questions. **20**

31. Two of the vertices of a triangle are (1, –6) and (–5, 2). The centroid of the triangle is (–2, 1). Find the third vertex of the triangle. Also find the area of this triangle.

    OR

    Prove that not both co-ordinates of all the vertices of an equilateral triangle can be rational numbers.

32. Prove that $y = x + 3$ is a tangent to the Parabola $y^2 = 12x$. Also find co-ordinates of its point of contact.

33. If the difference between measures of the eccentric angles of P and Q is $\pi/2$ and

    if $\overleftrightarrow{PQ}$ cuts intercepts c and d on the axes,

    then prove that $\dfrac{a^2}{c^2} + \dfrac{b^2}{d^2} = 2$, where P and Q are on the Ellipse.

34. For the rectangular Hyperbola $x^2 - y^2 = 9$, consider the tangent at (5, 4). Find the area of the triangle, which this tangent makes with two asymptotes.

35. Find the point of intersection of the lines

    L: $\dfrac{x-3}{1} = \dfrac{y+2}{-1} = \dfrac{z+1}{-1}$ and M: $\dfrac{x}{2} = \dfrac{z+3}{3}$; $y = -1$.

36. Get the radius and the centre of the Circle that is formed by the intersection of Sphere $x^2 + y^2 + z^2 = 25$ and the Plane $2x + 2y + z = 12$.

37. For $y = (\tan x)^x + x^{\tan x}$, find $\dfrac{dy}{dx}$.

38. Curves $x^2 y = 1$ and $a^5 y = x^3$ are intersecting perpendicularly. Prove that $a^6 = 6$.

39. If initial velocity of Projectile is 28 m/s. and horizontal range is 40 m., then find measure of angle of Projection.

OR

When acceleration is constant and instantaneous speed is 22 m/s, the particle covers a distance of 10320 m. in 60 seconds, then find the acceleration.

40. Obtain $\int \tan^3 x \, dx$.

OR

Obtain $\int \dfrac{x + \sin x}{1 + \cos x} \, dx$.

SECTION - D

Answer the following questions from S.Nos. 41 to 50.

Each question carries THREE marks. Answer as directed in the questions.   **30**

41. For A(1, 2) and B(2, 1), find the point dividing $\overline{AB}$ from A's side in n equal parts. From this, find the coordinates of the point of trisection.

42. Find the equation of the Circle which passes through (1, –2) and (4, –3) and having a centre lies on the line $3x + 4y = 7$.

OR

Find the equation of the Circle, if common chord of the circles $x^2 + y^2 - 4x = 0$ and $x^2 + y^2 - 6y = 0$ as the diametre.

43. Find a unit vector in R^3 making an angle of measure $\pi/3$ with each of the vector (1, –1, 0) and (0, 1, 1).

44. Prove that centroid and incentre for the equilateral triangle are same. With this, if A(6, 4, 6), B(12, 4, 0) and C(4, 2, –2), then find the incentre for triangle ABC by using vectors.

45. Find the foot of the perpendicular and equation of perpendicular line passing through (2, –1, 2) to plane $2x - 3y + 4z = 44$.

OR

Find the equation of the Plane passing through the line of intersection of the planes $3x - 4y + 5z = 10$ and $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

46. A water tank is in the form of an inverted cone. The radius and the height of the cone are 10 cm. and 20 cm. respectively. If water is poured in the tank at the rate of 5 cm^3 / second, then how fast the height of the water will be increasing when its depth is 15 cm. ?

OR

Prove that the length of the part of the tangent intercepted between the axes coordinate of the Curve $x^{2/3} + y^{2/3} = a^{2/3}$ is constant. $(a > 0)$

47. Evaluate : $\displaystyle\int_{-1}^{2} |2x - 1| \, dx$.

48. Find the area of the region bounded by curves $y = x^2$ and $y = x + 2$.

OR

Find the area of the region bounded by curves $y^2 = 4x$ and $x^2 = 4y$.

49. Evaluate $\dfrac{dy}{dx} + \dfrac{2y}{x} = e^x$.

50. Find $\displaystyle\lim_{x \to 1} \dfrac{a^x - (a+1)^x + 1}{x - 1}$.

SECTION - E

Answer the following questions from S.Nos. 51 to 54.

Each question carries FIVE marks.   20

51. Find the equation of a circum-circle of the triangle formed by the lines
    $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

OR

Find the co-ordinates C and D for the square ABCD,

if A(−1, 3) and B(2, −2).

52. Find : $\lim\limits_{x \to -1^+} \dfrac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$

53. If $2x = y^{1/m} + y^{-1/m}$; $x \geq 1$,

    then prove that $\left(x^2 - 1\right) y_2 + xy_1 = m^2 y$.

54. Evaluate :

    $\int \cos 2x \cdot \cos 4x \cdot \cos 6x \ dx$.

    ## OR

    Evaluate :

    $\int (4x + 3) \sqrt{x^2 - 4x + 13} \ dx$.