SECTION-A

10 × 2 = 20

VERY SHORT ANSWER TYPE QUESTIONS

Note : Attempt all questions. Each question carries 2 marks.

1. Find the equation of the line perpendicular to $5x - 3y + 1 = 0$ and passing through $(4, -3)$.

2. Find the angle between the lines $2x + y + 4 = 0, y - 3x = 7$.

3. Find the distance between the point $P(3, -1, 2)$ and the midpoint of the line segment joining the points $A\,(6, 3, -4), B\,(-2, -1, 2)$.

4. Find the equation of the plane passing through $(0, 0, -4)$ and perpendicular to the line joining the points $(1, -2, 2)$ and $(-3, 1, -2)$.

5. Compute $\underset{x \to a}{Lt} \dfrac{\sin(x-a) \cdot \tan^2(x-a)}{(x^2 - a^2)^2}$.

6. Show that $\underset{x \to \infty}{Lt} \{\sqrt{x^2 + x} - x\} = \dfrac{1}{2}$.

7. Check the continuity of the function f given by $f(x) = \begin{cases} x+1 & \text{if} \quad x \le 1 \\ 2x & \text{if} \quad 1 < x < 2 \\ 1+x^2 & \text{if} \quad x \ge 2 \end{cases}$ at 1 and at 2.

8. Find the derivative of $Tan^{-1}\sqrt{\dfrac{1-x}{1+x}}$ with respect x.

9. Find Δy and dy for the function $y = e^x$, when $x = 0$ and $\Delta x = 0{\cdot}1$.

10. Show that the equation of the tangent to the curve $(x/a)^n + (y/b)^n = 2\ (a \ne 0, b \ne 0)$ at the point (a, b) is $x/a + y/b = 2$.

SECTION-B

5 × 4 = 20

SHORT ANSWER TYPE QUESTIONS

Note : Answer any *FIVE* questions. Each question carries 4 marks.

11. $A(5, 3)$ and $B(3, -2)$ are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq. units.

12. Show that the axes are to be rotated through an angle of $\frac{1}{2} Tan^{-1}\left(\frac{2h}{a-b}\right)$ so as to remove the xy term from the equation $ax^2 + 2hxy + by^2 = 0$.

13. Find the equation of the straight line making equal intercepts on the co-ordinate axes and passing through the point of intersection of the lines $2x - 5y + 1 = 0$ and $x - 3y - 4 = 0$.

14. If $\sin y = x \sin (a + y)$ then show that $\frac{dy}{dx} = \frac{\sin^2 (a+y)}{\sin a}$.

15. If $y = ax^{n+1} + bx^{-n}$, then show that $x^2 y'' = n(n+1) y$.

16. Sand is poured from a pipe at the rate of 12 cc/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand-cone increasing when the height is 4 cm ?

17. If $u = Tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, show that $x \cdot u_x + y \cdot u_y = \sin 2u$.

SECTION-C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Note : Answer any *FIVE* questions. Each question carries 7 marks.

18. Find the circumcentre of the triangle whose sides are $3x - y - 5 = 0$, $x + 2y - 4 = 0$ and $5x + 3y + 1 = 0$.

19. Find the condition for the chord $lx + my = 1$ of the circle $x^2 + y^2 = a^2$ to subtend a right angle at the origin.

20. Show that the angle θ between the pair of lines $ax^2 + 2hxy + by^2 = 0$ is given by $\cos\theta = \left|\frac{a+b}{\sqrt{(a-b)^2 + 4h^2}}\right|$.

21. Find the angle between the two lines whose direction cosines are given by the relations $6bc + 5ab - 2ca = 0$ and $3a + b + 5c = 0$.

22. If $f(x) = Sin^{-1}\sqrt{\frac{x-\beta}{\alpha-\beta}}$ and $g(x) = Tan^{-1}\sqrt{\frac{x-\beta}{\alpha-x}}$, then show that $f'(x) = g'(x)$.

23. Show that the curves $y^2 = 4(x+1)$ and $y^2 = 36(9-x)$ intersect orthogonally.

24. Find the point at which the function $f(x) = \sin x\,(1 + \cos x)$ has maximum value.