SECTION - A

VERY SHORT ANSWER TYPE QUESTIONS $10 \times 2 = 20$

(Attempt 'ALL' questions. Each question carries '2' marks)

1. If the product of the intercepts made by the straight line $x \tan\alpha + y \sec\alpha = 1$ $(0 \leq \alpha < \pi/2)$ on the coordinate axes is equal to $\sin\alpha$, find α..

2. Find the value of p, if the lines $3x + 4y = 5$; $2x + 3y = 4$; $px + 4y = 6$ are concurrent.

3. Find the coordinates of the vertex 'C' if in triangle ABC, its controid is the origin and the coordinates of A, B are $(1, 1, 1)$ and $(-2, 4, 1)$ respectively.

4. Find the equation of the plane passing through the points $(1, 1, 1)$, $(1, -1, 1)$ and $(-7, -3, -5)$. Also show that it is parallel to y-axis.

5. Find $\underset{x \to 0}{Lt} \dfrac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$.

6. Find $\underset{x \to 1}{Lt} \dfrac{\sin(x-1)}{(x^2-1)}$.

7. If $f: R \to R$ is a function such that $f(x+y) = f(x) + f(y)$, $\forall\ x, y \in R$ and if f is continuous at a single point in R then show that f is continuous on R.

8. Find the derivative of $\log\left(\dfrac{x^2+x+2}{x^2-x+2}\right)$ with respect to 'x'.

9. Find δf and df if $f(x) = x^2$, $x = 20$, $\delta x = 0{\cdot}1$

10. Show that in the curve $xy = a^2$ the subtangent varies as the abscissa.

SECTION - B

SHORT ANSWER TYPE QUESTIONS $5 \times 4 = 20$

(Attempt any 'FIVE' questions. Each question carries '4' marks)

11. Find the equation of locus of a point the difference of whose distance from $(-5, 0)$ and $(5, 0)$ is 8 units.

12. If the transformed equation of a curve is $17X^2 - 16XY + 17Y^2 = 225$ when the axes are rotated through an angle 45°, then find the original equation of the curve.

13. If the acute angle between the lines $4x - y + 7 = 0$, $kx - 5y - 9 = 0$ is 45°, then find the value of k.

14. Find the derivative of the function $\frac{1}{x^2+1}$ from the first principle.

15. Find derivative of $Tan^{-1}(\sec x + \tan x)$ with respect to x.

16. A man 180 cm high, walks at a uniform rate of 12 km per hour away from a lamp post of 450 cm high. Find the rate at which the length of his shadow increases.

17. If $u = Tan^{-1}\left(\frac{x^3 - y^3}{x^3 + y^3}\right)$, then show that $xu_x + yu_y = 0$.

SECTION - C

LONG ANSWER TYPE QUESTIONS **5 × 7 = 35**

(Attempt any 'FIVE' questions. Each question carries '7' marks)

18. Find the circumcentre of the triangle whose vertices are (1, 3), (0, – 2) and (– 3, 1).

19. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel straight lines, then prove that i) $h^2 = ab$ ii) $af^2 = bg^2$ and the distance between the parallel lines $= \frac{2\sqrt{g^2 - ac}}{a(a+b)} = \frac{2\sqrt{f^2 - bc}}{b(a+b)}$.

20. Show that the equation $2x^2 - 13xy - 7y^2 + x + 23y - 6 = 0$ represents a pair of straight lines. Find the point of intersection and the acute angle between them.

21. Find the direction cosines (l, m, n) of the lines which are connected by the relations $l + m + n = 0$, $2lm - mn + 2nl = 0$. Also find the acute angle between the lines.

22. If $x^y + y^x = a^b$ then show that $\frac{dy}{dx} = -\left(\frac{y\,x^{y-1} + y^x \log y}{x^y \log x + x\,y^{x-1}}\right)$.

23. If the tangent at any point P on the curve $x^m y^n = a^{m+n}$ $(mn \neq 0)$ meets the coordinate axes in A, B, then show that $AP : BP$ is constant.

24. Show that the semi-vertical angle of the right circular cone of a maximum volume and of given slant height is $Tan^{-1}(\sqrt{2})$.