S.E. (Chemical) (Sem III) (Rev), Dec, 07.

Sub: Applied Mathematics - III 8/1/08

Lib

Con. 5762-07.

(REVISED COURSE)

CD- 6768

( 3 Hours)

[Total Marks : 100

N. B. : (1) Question No. 1 is **compulsory**.

(2) Attempt any **four** questions from the remaining **six**.

(3) **All** questions carry **equal** marks.

(4) Use of **Calculator's (Non-Programmable)** is allowed.

1. (a) If $f(t) = t + 1 \quad 0 < t < 2$

$= 3 \quad t > 2$

Find $L[f(t)]$ and $L[f'(t)]$

(b) Prove that :

$$J_{-5/2}(x) = \sqrt{\frac{2}{\Pi x}}\left[\frac{3}{x}\sin x + \frac{3-x^2}{x^2}\cos x\right]$$

(c) Under the mapping $w = \frac{1}{z}$ show that the image of the hyperbola $x^2 - y^2 = 1$ is hemniscale $\rho^2 = \cos 2\phi$.

(d) Using milne predictor-corrector formula. Find $y(1\cdot4)$ given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$, $y(1\cdot1) = 1\cdot233$, $y(1\cdot2) = 1\cdot548$, $y(1\cdot3) = 1\cdot979$.

2. (a) Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is +ve integer.

(b) Prove that :

L [f(t)] and L [f'(t)]

$$L\left[\sin h\frac{1}{2}t \sin\frac{\sqrt{3}}{2}t\right] = \frac{\sqrt{3}}{2}\left(\frac{s}{s^4+s^2+1}\right)$$

(c) Using modified Euler's method. Find $y(0\cdot1)$ and $y(0\cdot2)$ given that $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$

(d) If f(z) is analytic function. Show that

$$\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$$

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4. (a) A pilot plant reactor was charged with 50 kg napthalene and 200 kg (98% by mass) $H_2So_4$. The reaction goes to near completion. The product distribution was found to be 18·6%

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3. (a) Prove that : $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$

(b) Prove that : $J_0'''(x) = \frac{1}{x} J_0(x) + \left(\frac{2}{x^2} - 1\right) J'(x)$

(c) Find the bilinear transformation which maps the points z = 2, i, −2 on to the points w = 1, i, −1 resp. Also find fixed pts.

(d) Using Runge-Kutta method of 4th order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given y(0) = 1 at x = 0·2

4. (a) Using Laplace transform solve $\frac{d^4y}{dt^4} + \frac{d^2y}{dt^2} - 2y = 0$ given y(0) = 0, $y'(0) = -1, y''(0) = 0, y'''(0) = 1$

(b) Evaluate $\int_c \frac{z+2}{z^3 - 2z^2} dz$ where C is $|z - 2 - i| = 2$

(c) Prove that : $\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x)\right] = 2\left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x)\right]$

(d) Find y(0·1) from the system of equation

$\frac{dy}{dx} = z, \frac{dz}{dx} = -xz - y$ given $y(0) = 1, y'(0) = 0$ using R.K. 4th order.

5. (a) Evaluate : $\int x^3 J_3(x) dx$

(b) Evaluate : $\int_c \frac{z^2}{(z-1)^2 (z+1)} dz$ where C is $|Z| = 2$ using residue theorem.

(c) Find $L^{-1}\left[\frac{5s^2 + 8s - 1}{(s+3)(s^2+1)}\right]$

(a) Using convolution theorem find

$$L^{-1}\left[\frac{s^2}{s^4+13s^2+36}\right]$$

(b) Evaluate : $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos\theta + a^2}\, d\theta$ where $-1 < a < 1$

(c) Prove that : $J_{n+1}(x) = x \int_0^1 J_n(xy)\, y^{n+1}\, dy$

(d) Using Euler's method Solve $\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8{\cdot}5$ from $x = 0$ to $x = 4$ using step size of 0·5 the initial condition at $x = 0$ is $y = 1$.

) Find $L^{-1}\left[\frac{1}{s}\log\left(\frac{s+1}{s+2}\right)\right]$

) Show that general solution of equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(3 - \frac{1}{4x^2}\right)y = 0$ is

$$y = A\, J_{1/2}\left(\sqrt{3}\, x\right) + B\, J_{-1/2}\left(\sqrt{3}\, x\right)$$

Using Taylor's series method find y at x = 0·1, 0·2 given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ (Correct to 4–decimal places)

Show that $u = \left(r - \frac{a^2}{r^2}\right)\sin\theta$ can not be real part of any analytic function.

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