4/1/07. (REVISED COURSE) (95)

(3 Hours) [Total Marks : 100

: (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Non-programmable** calculator is **allowed.**

(a) Prove that :– 5

$$L[\sin h^5 t] = \frac{5!}{(s^2-1)(s^2-9)(s^2-25)}$$

(b) Find the constant a, b, c, d, e, if– 5

$f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic.

(c) Prove that : 5

$$I_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\ \sin h\, x$$

(d) Is $A = \begin{bmatrix} \frac{2+i}{3} & \frac{2i}{3} \\ \frac{2i}{3} & \frac{2-i}{3} \end{bmatrix}$ unitary matrix ? 5

(a) Prove that : 6

$$\int_0^1 x(1-x^2)J_0(ax)\,dx = \frac{4}{a^3}J_1(a) - \frac{2}{a^2}J_0(a)$$

(b) Find– 6

(i) $L[\sin 2t \cdot \cos t \cdot \cos h\, 2t]$

(ii) $L^{1}\left[\frac{s+2}{s^2(s+3)}\right]$

(c) Find the non-singular matrix P and Q such that PAQ is in normal form. Hence find rank A also find A^{-1} 8

$$\text{where } A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(a) Prove that : 6

$$\int_0^\infty \frac{e^{-\sqrt{2}\,t}\sin t \sin ht}{t}dt = \frac{\pi}{8}$$

(b) Determine the analytic function f(z) given $3u + 2v = y^2 - x^2 + 16xy$ 6

(c) Prove that :– 8

(i) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right]$

(ii) $\int_0^1 x^{5/2} J_{3/2}(ax)\,dx = \frac{1}{a}J_{5/2}(a)$

[TURN OVER

$$\text{adj } A = \begin{bmatrix} -2 & 1 & 3 \\ -2 & -3 & 11 \\ 2 & 1 & -5 \end{bmatrix}$$

(b) Find :–

$L^{-1}\left[\dfrac{1}{(s-3)(s+4)^2}\right]$ using convolution theorem.

(c) Under the mapping $w = \dfrac{1}{z}$ show that the image of–

(i) the circle $|z - 3i| = 3$ is the line $6v + 1 = 0$

(ii) the hyperbola $x^2 - y^2 = 1$ is lemniscate $R^2 = \cos 2\phi$.

5. (a) Prove that $y = x^{-n} J_n(x)$ is a solution of $x^2 \dfrac{d^2y}{dx^2} + (1+2n)\dfrac{dy}{dx} + xy = 0$.

(b) Find the matrix A satisfying the equation–

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$$

(c) Solve : $\dfrac{d^2y}{dt^2} + 4y = H(t-2)$ with $y(0) = 0$, $y'(0) = 1$

6. (a) Find the Bilinear transformation which maps the pt's $z = 1, i, -1$ in to the po
$w = i, 0, -i$. Also find fixed points.

(b) Find the constant a, b, c if A is orthogonal where $A = \dfrac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$.

(c) (i) If $L\left[\text{erf}\sqrt{t}\right] = \dfrac{1}{s\sqrt{s+1}}$ then find $L\left[t \text{ erf } 3\sqrt{t}\right]$

(ii) Find $L^{-1}\left[\dfrac{1}{s}\log\left(1 + \dfrac{1}{s^2}\right)\right]$

7. (a) Show that the function $u = \dfrac{1}{2}\log(x^2 + y^2)$ is harmonic and find it's harmonic conju

(b) Determine value of k, do the equation–

$x + y + z = 1$
$x + 2y + 4z = k$
$x + 4y + 10z = k^2$

have a solution. Solve them completely.

(c) Prove that : (i) $3 J_6(\sqrt{30}) + 5 J_2(\sqrt{30}) = 0$

(ii) $\dfrac{d}{dx}\left(x J_n J_{n+1}\right) = x\left(J_n^2 - J_{n+1}^2\right)$