2) Attempt any four questions out of remaining six questions.
3) Non-programmable calculator is allowed.

a) Find L [t cost (wt − α)] where w and α are constant. 5

b) Under the transformation $w = \frac{z-1}{z+1}$ ST. the map of the straight line x = y is a circle and find it's centre and radius. 5

c) P.T. $I_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \cosh x$ 5

d) Using Adam's predictor corrector method find y(0·4) given $\frac{dy}{dx} = xy + y^2$ with y(0) = 1 y(0·1) = 1·1169, y(0·2) = 1·2774, y(0·3) = 1·5041. 5

a) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. 6

b) If $L[t \sin at] = \frac{2as}{(s^2+a^2)^2}$ then Evaluate 6

(i) L(at cosat + Sin at] (ii) L(2 cosat − at sin at]

a) Solve : $\frac{dy}{dx} + y + xy^2 = 0$ with y(0) = 1 over [0, 0·2] (take h = 0·1) by Runge Kutta method of 4th order [correct to 4-decimal places] 8

b) Using Laplace transform solve : 6

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1 \quad \text{given } y = 0,\ y^1 = 0 \text{ at } t = 0.$$

a) Find the orthogonal trajectories of the family of curve represented by equation $x^3y - xy^3 = a$. 6

b) Prove that— 8

(i) $J_5''(x) = \left(\frac{30}{x^2} - 1\right) J_5(x) - \frac{1}{x} J_0(x)$

(ii) $\frac{d}{dx} x^{n/2} J_n(\sqrt{x}) = \frac{1}{2} x^{\frac{n-1}{2}} J_{n-1}(\sqrt{x})$.

a) Using modified Euler's method find y(0·2) and y(0·4) given $\frac{dy}{dx} = n + \sqrt{y}$ with y(0) = 1. 6

b) Evaluate $\int_c \frac{z+3}{z^2+2z+5} dz$ where c is | z + 1 − i | = 2. 6

c) Find (i) $L^{-1}\left[\frac{3s}{(2s+1)^2}\right]$ 8

(ii) $L^{-1}\left[\cot^{-1} 2/s^2\right]$

[TURN OVER

5. (a) Show that, the transformation $y = \frac{u}{\sqrt{x}}$ a Bessel's differential equation reduces to

$$\frac{d^2u}{dx^2} + \left[1 - \frac{n^2 - \frac{1}{4}}{x^2}\right]u = 0$$

(b) Find the Bilinear transformation which maps the points $z = 1, -i, -1$ into the points $i, \infty, -i$ and hence find the image of $|z| > 1$.

(c) Find $y(0\cdot1)$ and $z(0\cdot1)$ form the equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with $y(0) = 1$, $y'(0) = 1$ By using Runge Kutta 4[th] order.

6. (a) Prove that :— $\int_0^x x^{-n} J_{n+1}(x)dx = \frac{1}{2^n\ n+1} - x^{-n} J_n(x)$

(b) Evaluate : $\int_{-\infty}^{\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9} dx$ using contour integration.

(c) State convolution theorem and find

$$L^{-1}\left[\frac{s+3}{\left(s^2 + 6s + 13\right)^2}\right]$$

7. (a) Evaluate : —

$$\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} dt$$

(b) Show that : $\int x J_{2/3}\left(x^{3/2}\right) dx = \frac{-2}{3} x^{\frac{1}{2}} J_{-\frac{1}{3}}\left(x^{\frac{3}{2}}\right) + c$

(c) Evaluate using Residue theorem

(i) $\int_C \frac{(z+4)^2}{z^4 + 5z^3 + 6z^2} dz$ where C [$|z| = 1$.

(ii) Find the Residue of $f(z) = \frac{1 - e^{-2z}}{z^4}$ at it's pole.