SE (Chemical) (Sem. III)(R) May 09
Applied Maths III
8/6/09

134

n. 2878-09. VR-3372

**(REVISED COURSE)**

(3 Hours) **[Total Marks : 100**

**B. :** (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** remaining **six** questions.
(3) Non-programable **calculator** is **allowed.**

1. (a) Given $L\left[2\sqrt{\frac{t}{\pi}}\right] = \frac{1}{s^{3/2}}$ then prove that $L\left[\frac{1}{\sqrt{\pi t}}\right] = \frac{1}{\sqrt{s}}$. **5**

(b) Prove that— **5**

$$J_2^{1}(x) = \left(1 - \frac{4}{x^2}\right) J_1(x) + \frac{2}{x} J_0(x)$$

(c) Find the image of $|z| = 1$ under the transformatoin $w = 2z + z^2$. **5**

(d) If $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ verify that $A\,(\text{adj}A) = |A|\,I$. **5**

2. (a) Prove that— **6**

$$\int_0^{\infty} \frac{\sin 2t + \sin 3t}{t\,e^t}\,dt = \frac{3\pi}{4}.$$

(b) Prove that— **6**

$$\int x\, J_{2/3}\left(x^{3/2}\right) dx = -\frac{2}{3} x^{1/2}\, J_{-1/3}\left(x^{3/2}\right) + c.$$

(c) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ then prove that $(I - N)(I + N)^{-1}$ is a unitary matrix. **8**

3. (a) Find the analytic function f(z) whose imaginary part is $e^{-x}(y\cos y - x\sin y)$. **6**

(b) Find $L\left[\text{erf}\sqrt{t}\right]$. **6**

(c) Prove that— **8**

$$4\,J_n''(x) = J_{n-2}(x) - 2\,J_n(x) + J_{n+2}(x)$$

using this hence P.T.

$$8J_0'''(x) + 2J_3(x) + 6J_0'(x) = 0$$

**[TURN OVER**

time t

V-Ex-1-09-G-Scan-65



4. (a) If $s = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $A = \frac{1}{2}\begin{bmatrix} 4 & -1 & 1 \\ -2 & 3 & -1 \\ 2 & 1 & 5 \end{bmatrix}$

Prove that $SAS^{-1} = \text{dia.}\ (2, 3, 1)$.

(b) Find $L^{-1}\left[\frac{1}{s\sqrt{s+4}}\right]$ using convolution theorem.

(c) If f(z) is analytic in R, show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$

5. (a) Evaluate $\int x^3 J_3(x)\, dx$.

(b) Find the rank of matrix Reducing to normal form where $A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

(c) Solve : $(D^2 + 2D + 5)\, y = e^{-t} \sin t$ with $y(0) = 0,\ y'(0) = 1$.

6. (a) Find bilinear transformation which map's the points $z = 1, -i, -1$ in to the point's $w = i, \infty, -i$ and find the image of $|z| > 1$.

(b) Find $L^{-1}\left[\frac{s+2}{(s+3)(s+1)^3}\right]$.

(c) Test for consistency and solve them if consistent—

$$x - 2y + 3t = 2$$
$$2x + y + z + t = -4$$
$$4x - 3y + z + 7t = 8$$

7. (a) Determine $\alpha, \beta, \gamma$ if $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal.

(b) If f(z) is analytic with constant modulus then prove that f(z) is constant.

(c) Find $L^{-1}\left[\frac{1}{s}\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)\right]$

(d) Prove that—

$$J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{x}\sin x + \frac{3 - x^2}{x^2}\cos x\right]$$

———S———