S.E (Chemical) (Sem-III) (Rev.) Dec, 2010.
Sub: Applied Mathematics-III

Con. 6012-10. **GT-6300**

(3 Hours) **[Total Marks : 100**

N.B.: (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions from remaining **six** questions.
(3) **Non-programmable** calculator is **allowed**.

1. (a) Given $L\left[\sin\sqrt{t}\right] = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-\frac{1}{4s}}$ 5

Prove that $L\left[\frac{\cos\sqrt{t}}{\sqrt{t}}\right] = \sqrt{\frac{\pi}{s}}\, e^{-\frac{1}{4s}}$

(b) Prove that— 5

$$J_{5/2} = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right)$$

(c) Examine whether the vectors $X_1 = [3, 1, 1]$, $X_2 = [2, 0, -1]$, $X_3 = [4, 2, 1]$ are linearly independent. 5

(d) If $f(z) = r^2 \cos 2\theta + ir^2 \sin P\theta$ is analytic find P ? 5

2. (a) Prove that : 6

$$4J_n'''(x) + 3J_n'(x) + J_3(x) = 0$$

(b) Prove that— 6

$$\int_0^{\infty} \frac{e^{-\sqrt{2}t} \sinh t \cdot \sin t}{t}\, dt = \frac{\pi}{8}$$

(c) For what values of 'K' do the equation $x + y + z = 1$, $x + 2y + 4z = k$, $x + 4y + 10z = k^2$ have a solution. Slove them completely in each case. 8

3. (a) Find the analytic function f(z) such that : 6

$$u - v = (x - y)(x^2 + 4xy + y^2)$$

(b) Prove that : 6

$J_{-n}(x) = (-1)^n J_n(x)$ where n is +ve integer.

(c) Find :

(i) $L^{-1}\left[e^{-s}\left(\frac{1+\sqrt{s}}{s^3}\right)\right]$ 3

(ii) $L^{-1}\left[\tan^{-1}\left(\frac{2}{s^2}\right)\right]$ 5

4. (a) Show that every square matrix can be uniquely expressed as the sum of Hermitian matrix and skew Hermitian matrix. 6

(b) Find $L^{-1}\left[\frac{1}{s^3+1}\right]$. 6

(c) Under the mapping $w = \frac{1}{z}$ show that the image of : 8

(i) the circle $|z - 3i| = 3$ is the line $6v + 1 = 0$.
(ii) the hyperbola $x^2 - y^2 = 1$ is the lemniscate $R^2 = \cos 2\phi$.

5. (a) Is the following matrix orthogonal ? If not, can it be converted in to an orthogonal matrix ? If yes, how ? 6

where $A = \begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

(b) Solve using Lalpace transform : 6

$$\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t\,; \quad y(0) = 1$$

(c) Prove that : 8

$$\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x)\right] = 2\left[\frac{n}{x}J_n^2 - \frac{n+1}{x}J_{n+1}^2\right]$$

and hence deduce :

$$J_0^2 + 2\left(J_1^2 + J_2^2 + \ldots\right) = 1$$

6. (a) Using adjoint, find B such that— 6

$$AB = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 1 \\ 5 & 6 & 4 \end{bmatrix} \text{ if } A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}.$$

(b) Prove that— 6

$$\int J_5(x)dx = -J_4(x) - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x) + c$$

(c) Find the bilinear transformation which maps the points z = 2, i, −2 in to the points w = 1, i, −1 respectively. Also find fixed points. 8

7. (a) Show that $y = x\,J_n(x)$ is a solution of $x^2 - y'' - xy' + (1 + x^2 - n^2)\,y = 0$. 6

(b) If ϕ and ψ satisfies the Laplace equation then show that, s + it is analytic where— 6

$$s = \frac{\partial\phi}{\partial y} - \frac{\partial\psi}{\partial x}, \quad t = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial y}$$

(c) (i) If $L\left[\operatorname{erf}\sqrt{t}\right] = \dfrac{1}{s\sqrt{s+1}}$ 8

find $L\left[t \cdot \operatorname{erf} 3\sqrt{t}\right]$

(ii) Find $L\left[(1 + t\,e^{-t})^3\right]$