Total No. of Questions : 12] [Total No. of Printed Pages : 7

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F. E. Examination - 2009

ENGINEERING MATHEMATICS - I

(2003 Course)

Time : 3 Hours] **[Max. Marks : 100**

Instructions :

(1) Answer Q. 1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6 from section II and Q. 7 or Q.8, Q.9 or Q. 10, Q. 11 or Q.12 from section II.

*(2) Answer to the **two sections** should be written in **separate answer-books.***

(3) Figures to the right indicate full marks.

(4) Neat diagrams must be drawn wherever necessary.

(5) Use of non-propgrammable electronic pocket calculator is allowed.

(6) Assume suitable data if necessary.

SECTION - I

Q.1) (A) Reduce the following matrix to its normal form and hence find its rank :

$$\begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & 2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$ **[05]**

(B) Examine for consistency and solve them if consistent :

$2x + y + 2z + w = 6$

$6x - 6y + 6z + 12z = 36$

$4x + 3y + 3z - 3w = -1$ **[06]**

$2x + 2y - z + w = 10$

(C) Verify Cayley – Hamiltan Theorem and hence find A^{-1} for

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$ **[06]**

OR

Q.2) (A) Find Eigen Values and Corresponding Eigen Vectors for the matrix A :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ **[07]**

(B) Verify whether the matrix

$$A = \begin{bmatrix} 1/2 & 2/3 & 2/3 \\ 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \end{bmatrix}$$ is orthogonal. **[05]**

(C) Examine for linear dependence or independence for given vectors and if dependent, find the relation between them :

$X_1 = (1, 2, 3)$, $X_2 = (3, -2, 1)$ $X_3 = (1, -6, -5)$ **[05]**

Q.3) (A) Find the locus of Z satisfying $|Z - 3| - |Z + 3| = 4$ **[06]**

(B) Solve the equation

$x^7 - x^4 + x^3 - 1 = 0$ **[06]**

(C) Prove that :

$$\tan\left[i \log\left(\frac{a - ib}{a + ib}\right)\right] = \frac{2ab}{a^2 - b^2}$$ **[05]**

OR

Q.4) (A) Evaluate :

$(1 + i)^{100} + (1 - i)^{100}$ **[05]**

(B) If $\cos(u + iv) = x + iy$ **[06]**

prove that :

(i) $(1 + x)^2 + y^2 = (\cosh v + \cos u)^2$

(ii) $(1 - x)^2 + y^2 = (\cosh v - \cos u)^2$

(C) If z_1, z_2, z_3 represent vertices of an equilateral triangle, prove that :

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$ **[06]**

Q.5) (A) Find n^{th} derivative of $2e^x \cos x \cos 2x$. **[05]**

(B) If $y = (\sin^{-1} x)^2$

prove that :

$(1 - x^2)\, y_{n+2} - (2n + 1)\, x y_{n+1} - n^2 y_n = 0$ **[06]**

(C) Verify the Largange's Mean Value Theorem for $f(x) = 1-3x$ in $(1, 4)$. **[05]**

OR

Q.6) (A) If $y = \tan^{-1}\left(\dfrac{1 + x}{1 - x}\right)$

Find y_n. **[05]**

(B) Find n^{th} derivative of :

$x^2 e^x + x^2 \cos 3x$ **[06]**

(C) Verify Rolle's Theorem for :

$f(x) = x(x + 3)\, e^{x/2}$ in $[-3, 0]$. **[05]**

SECTION - II

Q.7) (A) Test for convergence the series :

$$1 + \frac{3}{4}x + \frac{5}{9}x^2 + \frac{7}{28}x^3 \;..... + \frac{(2n+1)}{n^3+1}x^n + ...$$ **[05]**

(B) Test for convergence of th series : **(Any One)** **[04]**

(1) $\frac{1}{1+5} + \frac{2}{1+5^2} + \frac{3}{1+5^3} + ...$

(2) $\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + ...$

(C) Attempt **any two** : **[08]**

(1) Expand $\log(1 + x + x^2 + x^3)$ upto a term in x^8.

(2) Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} \;.....\right)$$

(3) Expand $x^4 - 3x^3 + 2x^2 - x + 1$ in powers of $(x - 3)$.

OR

Q.8) (A) Test of convergence the service : **[05]**

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + ...$$

(B) Attempt **any one** : **[04]**

(1) Find whether the series

$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + ...$ is absolutely convergent.

(2) Test the convergence of the series

$$\frac{1}{3^P} + \frac{1}{5^P} + \frac{1}{7^P} + \frac{1}{9^P} + ...$$

(C) Attempt **any two** : **[08]**

(1) Prove that : $e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \ldots\ldots$

(2) Expand $\sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ in ascending powers of x.

(3) Prove that :

$$f(mx) = f(x) + (m - 1)\, x f'(x) + \frac{(m - 1)^2}{2!} x^2 f''(x) + \ldots$$

Q.9) (A) Attempt **any two** of the following : **[08]**

(1) Evaluate : $\lim_{x \to 0} \log_{\tan x} (\tan 2x)$

(2) Evaluate : $\lim_{x \to 2} \left[\frac{1}{x - 2} - \frac{1}{\log(x - 1)}\right]$

(3) Evaluate : $\lim_{x \to 0} \left[\frac{a^x + b^x}{2}\right]^{1/x}$

(B) If $v = \frac{1}{\sqrt{t}} e^{-x^2/4a^2 t}$ then show that $\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}$ **[05]**

(C) If $U = \sin^{-1}\left(x^3 + y^3\right)^{2/5}$ find $x^2 U_{xx} + 2xy\, U_{xy} + y^2 U_{yy}$. **[04]**

OR

Q.10) (A) Attempt **any two** of the following : **[08]**

(1) Evaluate : $\lim_{x \to e} (\log x)^{1/(1 - \log x)}$

(2) Evaluate : $\lim_{x \to 0} \frac{\log \sin x}{\cot x}$

(3) $\lim_{x \to a} \sqrt{\frac{a + x}{a - x}} \tan^{-1} \sqrt{a^2 - x^2}$ Evaluate.

(B) If $x = \frac{r}{2}(e^{\theta} + e^{-\theta})$, $y = \frac{r}{2}(e^{\theta} - e^{-\theta})$ then show that

$$\left(\frac{\partial x}{\partial r}\right)_{\theta} = \left(\frac{\partial r}{\partial x}\right)_{y}$$ **[04]**

(C) If $u = \sin(\sqrt{x} + \sqrt{y})$, **[05]**

prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}(\sqrt{x} + \sqrt{y}) \cos(\sqrt{x} + \sqrt{y})$$

Q.11) (A) A power dissipated in a resistor is given by $P = \frac{E^2}{R}$. Find the approximate percentage error in P when E is increased by 3% and R is decreased by 2%. **[05]**

(B) Find the extreme values of xy (a – x – y). **[06]**

(C) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = xy + yz + zx$ examine whether u, v, w are functionally dependent and if so find the relation between them. **[05]**

OR

Q.12) (A) Verify JJ' = 1 for the transformation

$x = uv, \; y = {}^{u}/_{v}.$ **[05]**

(B) If $u = xyz, \quad v = x^2 + y^2 + z^2, \quad w = x + y + z,$

find $\frac{\partial x}{\partial u}$. **[06]**

(C) As the deminsions of a triangle ABC are varied, show that the maximum value of cosA cosB cosC is obtained when the triangle is equilateral. **[05]**
