BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

June, 2006

CS-60 Ⓢ : FOUNDATION COURSE IN MATHEMATICS IN COMPUTING

Time : 3 hours *Maximum Marks : 75*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***three*** *questions from Q. no. 2 to 5. Calculators are not allowed.*

1. (a) Find the maximum possible domain of the function f, defined by $f(x) = \sqrt{\dfrac{x}{x^2 - 9}}$. *2*

(b) Which of the following statements are true ? Give reasons for your answer. *10*

(i) The direction ratios of $\dfrac{x-1}{2} = \dfrac{y-3}{3}$, $z = 7$ are 2, 3, 1.

(ii) All the planar sections of a hyperboloid are hyperbolas.

(iii) $f(x) = \cos x + \sin x$ is an odd function.

(iv) $\{1, \phi, \text{IGNOU}\}$ is a set.

(v) $ax^3 + bx^2 + cx + d = 0$, a, b, c, d $\in$ **R** has three roots in **R**.

(c) Find the length of the major and minor axes, and the eccentricity of $3x^2 - 4y^2 = 12$. 3

(d) Find $\frac{d}{dx}\left[\int_{5}^{x^2(1-x^2)} \sin^{-1}(\cos 5t)\,dt\right]$. 2

(e) Taking four sub-divisions of the interval [0, 4], find an approximate value of $\int_0^4 \frac{x}{1+x^2}\,dx$ using the Trapezoidal rule. 3

(f) Can the following system of equations be solved by Cramer's rule ? If yes, apply the rule to solve it. Otherwise apply the Gaussian method to solve it

$2x - 3y + 4z = 5,$

$7x + 4 = y - 8z,$

$x + 8y - 4z + 19 = 0.$ 6

(g) If $y = \left(x - \sqrt{x^2 - 4}\right)^m$, then show that

$(x^2 - 4)\, y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2 - m^2)\, y_n = 0.$ 4

2. (a) Find all the points of continuity in **R** of the function f, defined by

$$f(x) = \begin{cases} -x^2 & \text{if } x \leq 0 \\ 5x - 4 & \text{if } 0 < x \leq 1 \\ 4x^2 - 3x & \text{if } 1 < x < 2 \\ 3x & \text{if } x \geq 2 \end{cases}$$ 4

(b) Evaluate

$$\int \frac{x^2 + 1}{1 + x^4} dx$$ 3

(c) Find the equation of a right-circular cylinder having for its base the curve $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 3

(d) Using Rolle's theorem, show that there is $\theta \in]-1, 1[$ such that $\sin 2\theta = -4\theta^3$. 5

3. (a) Prove that

$$\int_0^{\pi/2} \frac{\cos^n x}{\cos^n x + \sin^n x} dx = \frac{\pi}{4}$$ 3

(b) Find the volume of the solid generated by the revolution of an arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about the x-axis. 4

(c) Solve the equation

$$x^4 - 4x^2 + 8x + 35 = 0,$$

given that one of the roots is $2 + i\sqrt{3}$. 4

(d) For which values of $\lambda \in \mathbf{R}$ does the plane $\Pi \equiv x + y + z = \lambda$ touch $S \equiv x^2 + y^2 + z^2 = 1$? Also find the points of contact of those planes Π that touch S. 4

4. (a) Find all the 5^{th} roots of $5i - 2$. 4

(b) Find all the asymptotes of the curve $(x^2 - 7x + 6)\, y = x^2 + 3x - 1$. 4

(c) Prove that the cone

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

possesses three mutually perpendicular tangent planes if and only if $bc + ca + ab = f^2 + g^2 + h^2$. 4

(d) Find the upper and lower product sums of f, defined by $f(x) = \frac{3}{x}$, with respect to the partition $P = \{1, 3, 5, 7\}$ of $[1, 7]$. 3

5. (a) If $a > b > 0$, and $n \in \mathbf{N}$, show that

$$a^{n-1} + ba^{n-2} + \ldots + b^{n-1} > n(ab)^{\frac{n-1}{2}}.$$

Hence prove that

$$a^n - b^n > n(ab)^{\frac{n-1}{2}}(a - b).$$

4

(b) Reduce the equation

$$x^2 - 3xy + y^2 - 6x + 6y - 2 = 0$$

to canonical form. Hence identify the conic it represents. Also draw a rough sketch of the conic given by the equation above. *8*

(c) Find an approximate value of $(0 \cdot 98)^{5/2}$ using Maclaurin's series, upto 3 decimal places. *3*