Part III — STATISTICS

(New Syllabus)

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 150

PART – I

Note : i) Answer *all* the questions.

ii) Each question carries *one* mark.

Choose the correct answer : 50 × 1 = 50

1. When a coin and a die are thrown, the number of all possible cases is

    a) 7 b) 8

    c) 12 d) 6.

2. Probability can take values from

    a) – ∞ to ∞ b) – 1 to 1

    c) – ∞ to 1 d) 0 to 1.

3. When three dice are thrown, the probability of sum being 18 is

    a) $\dfrac{1}{216}$ b) $\dfrac{2}{216}$

    c) $\dfrac{3}{216}$ d) 0.

4. If $P(A) = 0.5$, $P(B) = 0.3$ and the events A and B are independent then $P(A \cap B)$ is

    a) 0.8 b) 0.15

    c) 0.2 d) 0.015.

[Turn over

5. If $P(A) = 0.6$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$ then $P\left(\dfrac{A}{B}\right) =$

   a) $\dfrac{1}{7}$ b) $\dfrac{2}{7}$

   c) $\dfrac{3}{7}$ d) $\dfrac{1}{2}$.

6. The probability that a leap-year selected at random will contain 53 Wednesdays is

   a) $\dfrac{1}{7}$ b) $\dfrac{2}{7}$

   c) $\dfrac{3}{7}$ d) $\dfrac{1}{2}$.

7. If $P(A) = 0.15$, $P(B) = 0.25$ and $P(A \cap B) = 0.10$ then $P(A \cup B) =$

   a) 0.40 b) 0.10

   c) 0.30 d) 0.20

8. If $F(x)$ is distribution function then $F(+\infty) =$

   a) 0 b) 1

   c) -1 d) $-\infty$.

9. $E(2X + 3)$ is

   a) $E(2X)$

   b) $2E(X) + 3$

   c) $4E(X)$

   d) $2X + 3$.

10. Variance of the random variable X is

   a) $E(X^2) - [E(X)]^2$

   b) $[E(X)]^2 - E(X^2)$

   c) $E(X^2)$

   d) $[E(X)]^2$.

11. Variance of the random variable X is $\frac{1}{16}$, its standard deviation is

    a) $\frac{1}{16}$ b) $\frac{1}{4}$

    c) $\frac{1}{256}$ d) $\frac{1}{64}$

12. If $f(x)$ is the *p.d.f.* of the continuous random variable X, then $E\left(X^2\right) =$

    a) $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx$

    b) $\displaystyle\int_{-\infty}^{\infty} x\,f(x)\,dx$

    c) $\displaystyle\int_{-\infty}^{\infty} x^2\,f(x)\,dx$

    d) $\displaystyle\int_{-\infty}^{\infty} f(x^2)\,dx$.

13. If $F(x)$ is a cumulative distribution function of a continuous random variable X, then $F'(x) =$

    a) $f(x)$

    b) $2f(x)$

    c) 0

    d) $\frac{1}{2}f(x)$.

14. If $f(x)$ is the probability density function of a continuous random variable X with *p.d.f.* $f(x)$, then $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx =$

    a) 1 b) 0

    c) 0.5 d) 2.

15. Binomial distribution applies to

   a) rare events

   b) repeated alternatives

   c) 3 events

   d) impossible events.

16. The variance of a binomial distribution is

   a) np b) npq

   c) $\sqrt{npq}$ d) 0.

17. For a Poisson distribution

   a) Mean = Variance

   b) Mean < Variance

   c) Mean > Variance

   d) Mean $\neq$ Variance.

18. If the Poisson variables X and Y have parameters m_1 and m_2, then $(X + Y)$ is a Poisson variable with parameter

   a) $m_1 \cdot m_2$

   b) $m_1 - m_2$

   c) $m_1 + m_2$

   d) $m_1 \div m_2$

19. If the mean of a Poisson variable X is 1, then $P(X < 1) =$

   a) e^{-1} b) $1 - e^{-1}$

   c) $1 - \dfrac{5}{2} e^{-1}$ d) $1 - 2e^{-1}$.

20. Mode of the normal distribution is

   a) μ b) σ

   c) 0   d) $\dfrac{1}{\sqrt{2\pi}}$.

21. The area $P(0 < Z < \infty)$ is equal to

   a) 1   b) 0·1

   c) 0·5   d) 0.

22. If X is a normal variable with $\mu = 100$ and $\sigma^2 = 25$ then, $P(90 < X < 120) =$

   a) $P(-1 < Z < 1)$

   b) $P(-2 < Z < 3)$

   c) $P(-2 < Z < 4)$

   d) $P(4 < Z < 4·5)$.

23. The mean of the binomial distribution $15\,C_x \left(\dfrac{2}{3}\right)^x \left(\dfrac{1}{3}\right)^{n-x}$ is

   a) 5   b) 10

   c) 15   d) 1.

24. The standard error of the mean is

   a) σ^2 b) $\dfrac{\sigma}{n}$

   c) $\dfrac{\sigma}{\sqrt{n}}$ d) $\dfrac{\sqrt{n}}{\sigma}$.

25. The critical value of the test statistic at level of significance α for a two tailed test is

   a) $Z_{\frac{\alpha}{4}}$ b) $Z_{-\alpha}$

   c) $Z_{2\alpha}$ d) $Z_{\frac{\alpha}{2}}$

26. H_0 is rejected when H_0 is true, is

   a) Type II error

   b) Type I error

   c) simple error

   d) none of these.

27. The standard error of observed sample proportion P is

   a) $\sqrt{\dfrac{P(1-Q)}{n}}$

   b) $\sqrt{\dfrac{PQ}{n}}$

   c) $\sqrt{\dfrac{(1-P)Q}{n}}$

   d) $\dfrac{PQ}{n}$.

28. Large sample theory is applicable when

   a) $n > 30$

   b) $n < 30$

   c) $n < 100$

   d) $n < 1000$.

29. Statistic $Z = \left| \dfrac{\bar{x} - \bar{y}}{\sigma \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \right|$ is used to test the null hypothesis

   a) $H_0 : \mu_1 + \mu_2 = 0$

   b) $H_0 : \mu_1 - \mu_2 = 0$

   c) $H_0 : \mu = \mu_0$ (a constant)

   d) none of these.

30. Standard error of number of success is given by

   a) $\sqrt{\dfrac{pq}{n}}$ b) $\sqrt{npq}$

   c) npq d) $\sqrt{\dfrac{np}{q}}$.

31. If $n_1 = 60$, $n_2 = 140$, $x_1 = 40$, $x_2 = 80$, then $\hat{p}$ =

   a) $\dfrac{6}{10}$ b) $\dfrac{4}{10}$

   c) $\dfrac{5}{10}$ d) $\dfrac{2}{10}$.

32. t-distribution ranges from

   a) $-\infty$ to 0   b)   0 to ∞

   c) $-\infty$ to ∞ d)   0 to 1.

33. When observed and expected frequencies completely coincide, χ^2 will be

   a) -1 b)   0

   c)   1   d)   greater than 1.

34. The Yate's corrections are generally made when the number of degrees of freedom is

   a)   4   b)   5

   c)   < 5   d)   1.

35. Degrees of freedom for Chi-square in case of contingency table of order (4 × 3) are

   a)   12   b)   6

   c)   8   d)   9.

36. While testing the significance of the difference between the two samples means in case of small samples, the degrees of freedom is

a) $n_1 + n_2$

b) $n_1 + n_2 - 1$

c) $n_1 + n_2 - 2$

d) $n_1 + n_2 + 2$.

37. Customarily the larger variance in the variance ratio for F-statistic is taken

a) in the denominator

b) in the numerator

c) either way

d) none of these.

38. Equality of two population variances can be tested by

a) t-test

b) F-test

c) χ^2-test

d) none of these.

39. In the case of two-way classification with 120, 54, 45 respectively as TSS, SSC and SSE, then SSR is

a) 11

b) 21

c) 99

d) 219.

40. In the case of one-way classification with N observations and 't' treatments, the error degrees of freedom is

a) $N - 1$

b) $N - t$

c) Nt

d) $t - 1$.

41. Analysis of variance utilises

a) F-test
b) χ^2-test

c) z-test
d) t-test.

42. Number of components in time series is

a) 5
b) 4

c) 3
d) 2.

43. The sales of a departmental store on a certain festival time are associated with

a) secular trend

b) seasonal variation

c) irregular variation

d) none of these.

44. If the slope of the trend line is positive, it shows

a) Rising Trend

b) Declining Trend

c) Stagnation

d) none of these.

45. Given $N = 800$, $(A) = 160$, $(B) = 300$, $(AB) = 50$, the expected frequency A, B is

a) 60
b) 50

c) 80
d) 150.

46. If for the two attributes A and B, $(AB) > \dfrac{(A) \cdot (B)}{N}$, then the attributes are

a) independent

b) positively associated

c) negatively associated

d) no conclusion.

47. If A and B are independent, Yules' coefficient is equal to

a) 0   b) 1

c) -1 d) 0·5.

48. Decision theory is concerned with

a) the amount of information that is available

b) criteria for measuring the 'goodness' of a decision

c) selecting optimal decisions in sequencial problems

d) all of these.

49. Maximin return, maximax return and minimax regret are criteria that

a) lead to the same optimal decision

b) cannot be used with probabilities

c) both (a) and (b)

d) none of these.

50. Which of the following criteria does not apply to decision making under uncertainty ?

a) Maximin return   b) Maximax return

c) Minimax return   d) Maximize expected return.

PART – II

Note :   i)   Answer any *fifteen* questions.

ii)   Each question carries *two* marks.   15 × 2 = 30

51.   State the axioms of probability.

52.   When three coins are tossed, find the probability of getting 3 heads.

53.   A random variable X has the following distribution :

X	0	1	2	3	4	5	6	7	8
P (X)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find $P (X < 3)$.

54.   Verify whether $f(x) = \dfrac{2x}{9}$, $0 < x < 3$ is a probability density function.

55.   The probability density function of a continuous random vriable X is given by $f(x) = \dfrac{x}{2}$, for $0 < x < 2$. Find its mean.

56.   Find the binomial distribution whose mean is 3 and variance 2.

57.   Give any two examples of Poisson distribution.

58.   Define the normal distribution.

59.   Give the standard error of the difference of two sample proportions.

60.   Define Parameter and Statistic.

61.   In a test, if $Z_0 > Z_e$ what is the conclusion about the null hypothesis ?

62.   State any two properties of t-distribution.

63.   Define Chi-square variate.

64.   Define Degrees of freedom.

65.   Write any two different mehods of measuring trend.

[Turn over

66. What are the components of time series ?

67. Write the Yules' coefficient of association.

68. Verify whether the given data $N = 100$, $(A) = 75$, $(B) = 60$ and $(AB) = 15$ are consistent.

69. Write a note on decision tree.

70. What is a pay-off matrix ?

PART - III

Note : i) Answer any *six* questions.

ii) Each question carries *five* marks. $6 \times 5 = 30$

71. State and prove addition theorem on probabilities for not-mutually exclusive events.

72. A random variable X has the density function $f(x) = Ax^3$, $0 < x < 1$. Find A and mean.

73. With the usual notation, find p for binomial random variable X if $n = 6$ and 9. $P(X = 4) = P(X = 2)$.

74. If 2% of electric bulbs manufactured by a certain company are defective. Find the probability that in a sample of 200 bulbs more than 3 bulbs are defectve.

$\left[e^{-4} = 0.0183 \right]$

75. In a random sample of 400 persons from a large population 120 are females. Can it be said that males and females are in the ratio 5 : 3 in the population ? [Use 1% level of significance]

76. A random sample of size 20 from a population gives the sample standard deviation of 6. Test the hypothesis that the population standard deviation is 9.

77. Calculate the three yearly moving average of the following data :

Year	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
Production (in tonnes)	50	36	43	45	39	38	33	42	41	34

78. In a co-educational institution, out of 200 students, 150 were boys. They took an examination and it was found that 120 passed, 10 girls failed. Is there any association between sex and success in the examination ?

79. The pay-off table for three courses of action (A) with three states of nature (E) with their respective probabilities is given. Find the best course of action.

Events	E_1	E_2	E_3
Probability / Acts	0·2	0·5	0·3
A_1	2	1	1
A_2	3	2	0
A_3	4	2	1

PART - IV

Note : i) Answer any *four* questions.

  ii) Each question carries *ten* marks.   4 x 10 = 40

80. Three persons A, B and C are being considered for the appointment as the chairman for a company whose chance of being selected for the post are in the proportion 4 : 2 : 3 respectively. The probability that A, if selected will introduce democratization in the company structure is 0·3, the corresponding probabilities for B and C doing the same are respectively 0·5 and 0·8. What is the probability that the democratization would be introduced in the company ?

[Turn over

81. Four coins are tossed and number of heads noted. The experiment is repeated 200 times and the following distribution is obtained.

Number of heads :	0	1	2	3	4
Frequencies :	62	85	40	11	2

Fit a binomial distribution.

82. The means of two large samples of 1000 and 2000 items are 67·5 cm and 68·0 cm respectively. Can the sample be regarded as drawn from the population with standard deviation 2·5 cm ? Test at 5% level of significance.

83. Two random samples drawn from two normal populations are

Sample I :	20	16	26	27	22	23	18	24	19	25		
Sample II :	27	33	42	35	32	34	38	28	41	43	30	37

Test at 5% level of significance whether the two populations have the same variance.

84. A test was given to five students taken at random from the fifth class of three schools of a town. The individual scores are

School I	9	7	6	5	8
School II	7	4	5	4	5
School III	6	5	6	7	6

Carry out the analysis of variance.

85. Fit straight line trend by the method of least squares for the following data :

Year :	1983	1984	1985	1986	1987	1988
Sales : (Rs. in lakhs)	3	8	7	9	11	14

Also estimate the sales for the year 1992.

86. Consider the following pay-off (profit) matrix :

Action	States			
	(S_1)	(S_2)	(S_3)	(S_4)
A_1	5	10	18	25
A_2	8	7	8	23
A_3	21	18	12	21
A_4	30	22	19	15

No probabilities are known for the occurrence of the nature of states.

Compare the solutions obtained by each of the following criteria :

i) Maximin

ii) Laplace

iii) Hurwicz

(assume that $\alpha = 0.5$).