Part III — STATISTICS

(English Version)

llowed : 3 Hours] [Maximum Marks : 150

PART - I

Note : i) Answer *all* the questions.

ii) Each question carries *one* mark.

Choose the best answer : 50 × 1 = 50

1. The conditional probability of *B* given *A* is

a) $\dfrac{P(A \cap B)}{P(B)}$ b) $\dfrac{P(A \cap B)}{P(A)}$

c) $\dfrac{P(A \cup B)}{P(B)}$ d) $\dfrac{P(A \cup B)}{P(A)}$.

2. If *A* and *B* are independent events, then $P(A \cap B) =$

a) $P(A) + P(B)$ b) $\dfrac{P(A)}{P(A \cap B)}$

c) $\dfrac{P(A \cap B)}{P(B)}$ d) $P(A) . P(B)$

3. Probability can take values from

a) $-\infty$ to ∞ b) $-\infty$ to 1

c) 0 to 1 d) -1 to 1.

4. The probability of an event cannot be predicted before its actual happening. This is

   a) Statistical probability

   b) Mathematical probability

   c) a prior probability

   d) none of these.

5. $P(X) = 0.15$; $P(Y) = 0.25$, $P(X \cap Y) = 0.10$, then $P(X \cup Y) =$

   a) 0.10   b) 0.20

   c) 0.30   d) 0.40.

6. The probability of getting the total 11 when two dice are thrown is

   a) $\dfrac{1}{36}$ b) $\dfrac{11}{18}$

   c) $\dfrac{2}{18}$ d) $\dfrac{2}{36}$.

7. A coin is tossed five times. The number of points in the sample space is

   a) 12   b) 10

   c) 32   d) 64.

8. If $F(x)$ is the distribution function, then $F(-\infty)$ is

   a) -1 b) 0

   c) 1   d) ∞.

9. A random variable X has $E(X) = 2$ and $E\left(X^2\right) = 8$. Its variance is

   a) 4   b) 6

   c) 8   d) 2.

10. Variance of ($5x + 2$) is

a) $25\, Var\,(X)$

b) $5\, Var\,(X)$

c) $2\, Var\,(X)$

d) 25.

11. If $F(x)$ is a cumulative distribution function of a continuous random variable x with probability density function $f(x)$, then $F'(x)$ is

a) $f(x)$

b) $xf(x)$

c) $x^2 f(x)$

d) $f(x^2)$.

12. Mathematical expectation of a random variable X is also known as

a) standard deviation

b) variance

c) mode

d) mean.

13. In a discrete distribution function $p\left(x_j\right) =$

a) $F\left(x_j\right) - F\left(x_{j-1}\right)$

b) $F\left(x_j\right) + F\left(x_{j-1}\right)$

c) $F\left(x_{j-1}\right) - F\left(x_j\right)$

d) $F\left(x_j\right) \cdot F\left(x_{j-1}\right)$.

14. If $F(x) = \int_{-\infty}^{x} f(x)\, dx$, then $F(x)$ is

a) probability density function

b) probability mass function

c) discrete probability distribution function

d) distribution function of a continuous random variable.

15. The variance of a binomial distribution is

    a) npq b) np

    c) $\sqrt{npq}$ d) 0.

16. For a Poisson distribution

    a) mean > variance   b) mean = variance

    c) mean < variance   d) mean ≤ variance.

17. Poisson distribution is a limiting case of a binomial distribution when

    a) $n \to \infty$ $p \to 0$ $np = \sqrt{m}$

    b) $n \to 0$ $p \to \infty$ $p = \dfrac{1}{m}$

    c) $n \to \infty$ $p \to 0$ $np = m$

    d) $n \to \infty$ $p \to \infty$ $np = m$.

18. In a normal distribution skewness is

    a) one   b) zero

    c) greater than one   d) less than one.

19. $\left(\dfrac{2}{3} + \dfrac{1}{3} \right)^9$ refers to the binomial distribution and its standard deviation is

    a) $\sqrt{2}$ b) 2

    c) $\sqrt{4}$ d) 1.

20. If X is a normal variate with $\mu = 100$ and $\sigma^2 = 25$ then $P(90 < X < 120)$ is same as

    a) $P(-1 < Z < 1)$ b) $P(-2 < Z < 4)$

    c) $P(4 < Z < 4)$ d) $P(-2 < Z < 3)$.

21. If the mean of binomial distribution is 4 and the variance is 2 then the parameter is

    a) $\left(16, \dfrac{1}{2} \right)$ b) $\left(8, \dfrac{1}{12} \right)$

    c) $\left(16, \dfrac{1}{36} \right)$ d) $\left(8, \dfrac{1}{2} \right)$.

22. $P(-3 < Z < 3)$ takes the value

    a)   0·9973   b)   0·6587

    c)   0·6826   d)   0·3174.

23. In a normal distribution the first and third quartiles are equidistant from

    a)   median   b)   mean

    c)   mode   d)   all of these.

24. The standard error of the mean is

    a) σ^2 b) $\dfrac{\sigma}{n}$

    c) $\dfrac{\sigma}{\sqrt{n}}$ d) $\dfrac{\sqrt{n}}{\sigma}$.

25. Critical value of $|Z_\alpha|$ at 5% level of significance for two tailed test is

    a)   1·645   b)   2·33

    c)   2·58   d)   1·96.

26. Alternate hypothesis is

    a)   always left tailed

    b)   always right tailed

    c)   always one tailed

    d)   one tailed or two tailed test.

27. Testing $H_0 : \mu = 1500$ against $\mu < 1500$ leads to

    a) one sided left tailed test   b) one sided right tailed test

    c) two tailed test   d) none of these.

28. Standard error of number of success is given by

    a) $\sqrt{\dfrac{pq}{n}}$ b) $\sqrt{npq}$

    c) npq d) $\dfrac{\sqrt{np}}{q}$.

29. In a sample if .... , then it is called large sample.

    a) $n > 30$ b) $n < 30$

    c) $n \leq 30$ d) $n \geq 100$.

30. If $\hat{P} = \dfrac{2}{3}$, then $\hat{Q}$ is

    a) 1   b) $\dfrac{1}{3}$

    c) $\dfrac{2}{3}$ d) 0.

31. t-distribution ranges from

    a) $-\infty$ to 0   b) 0 to ∞

    c) $-\infty$ to ∞ d) 0 to 1.

32. While testing the significance of the difference between two sample means in case of small samples, the degrees of freedom is

    a) $n_1 + n_2$ b) $n_1 + n_2 - 1$

    c) $n_1 + n_2 - 2$ d) $n_1 + n_2 + 2$.

33. Equality of two population variance can be tested by

   a) *F*-test   b) *t*-test

   c) χ^2-test   d) Standard test.

34. The mean difference between 9 paired observations is 15·0 and the standard variation of differences is 5·0. Then the value of statistic *t* is

   a) 27   b) 9

   c) 3   d) 0.

35. Degrees of freedom in a contingency table is

   a) $(r+1)(c-1)$ b) $(r-1)(c-1)$

   c) $(r-1)(c+1)$ d) $(r+1)(c+1)$.

36. When observed and expected frequencies completely coincide, χ^2 will be

   a) -1 b) $+1$

   c) greater than 1   d) 0.

37. The Yate's corrections are generally made when the number of degrees of freedom is

   a) 5   b) < 5

   c) 1   d) 4.

38. Customarily the larger variance in the variance ratio for *F*-statistic is taken

   a) in the denominator   b) in the numerator

   c) in either way   d) none of these.

39. Analysis of variance technique originated in the field of

    a) agriculture   b) industry

    c) biology   d) genetics.

40. With 90, 35, 25 as TSS, SSR and SSC respectively in case of two-way classification, SSE is

    a) 50   b) 40

    c) 30   d) 20.

41. In the case of one-way classification with N observations and t treatments, the error degrees of freedom is

    a) $N-1$ b) $t-1$

    c) $N-t$ d) Nt.

42. A time series consists of

    a) two components   b) three components

    c) four components   d) five components.

43. The trend line obtained by the method of least square is known as the

    a) perpendicular line   b) asymptotes

    c) normal line   d) line of best fit.

44. Business forecasts are made on the basis of

    a) present data   b) past data

    c) policies and circumstances   d) all of these.

45. If $N = 500$, $(A) = 300$, $(B) = 250$ and $(AB) = 40$, the data are

    a) consistent   b) inconsistent

    c) positively placed   d) none of these.

46. If A and B are independent, Yule's coefficient is equal to

a) 1

b) 2

c) 3

d) 0.

47. In case of consistent data, no class frequency can be

a) negative

b) positive

c) zero

d) none of these.

48. Which of the following criteria does not apply to decision-making under uncertainty ?

a) Maximin return

b) Maximax return

c) Minimax return

d) Maximize expected return.

49. $H = \alpha$ (maximum pay-off in column) + (1 – α) (minimum pay-off in column) is

a) Savage's maximizes strategy

b) Hurwicz's maximizes strategy

c) EMV's maximizes strategy

d) Bayes' maximizes strategy.

50. Decision theory is concerned with

a) the amount of information that is available

b) criteria for measuring the goodness of a decision

c) selecting optimal decisions in sequential problems

d) all of these.

PART – II

Note : i)   Answer any *fifteen* questions.

ii) Each question carries *two* marks.   15 × 2 = 30

51. State the axioms of probability.

52. For two independent events A and B for which $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, find the probability that only one of them occurs.

53. Define probability density function.

54. When a die is thrown what is the expected value of outcome ?

55. Find the probability distribution of X when 2 coins are tossed, where X is defined as getting head.

56. For a binomial distribution, mean = 7 and variance = 16. Comment on it.

57. Give any two examples of Poisson distribution.

58. Write any two properties of normal distribution.

59. Define Standard Error.

60. State Null hypothesis.

61. What is the test statistic for difference between two means ?

62. Write any two properties of t-distribution.

63. Define Chi-square variate.

64. Write short note on Yate's correction.

65. What is cyclic variation ?

66. What are the methods of forecasting ?

67. What is Yule's coefficient of association ?

68. Explain consistency of the data.

69. Write briefly about Hurwicz criterion.

70. Find the minimax regret for the following pay-off table :

Act	States of Nature	
	S_1	S_2
A_1	10	15
A_2	20	12
A_3	30	11

PART – III

Note : i)   Answer any *six* questions.

ii)   Each question carries *five* marks.   6 × 5 = 30

71. An urn contains 5 red and 7 green balls. Another urn contains 6 red and 9 green balls. If a ball is drawn from any one of the two urns, find the probability that the ball drawn is green.

72. A random variable has the following probability distribution :

Values of X	0	1	2	3	4	5	6	7	8
P (X)	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i)   Determine the value of a

ii)   Find (a) $P (X < 4)$, (b) $P (1 < X < 5)$, (c) $P (X > 6)$.

[Turn over

73. 10% of the screws manufactured by an automatic machine are found to be defective. 20 screws are selected at random. Find the probability that at least 2 screws are defective.

74. If a random variable X follows Poisson distribution such that $P(X = 1) = P(X = 2)$, find (i) the mean of the distribution and

(ii) $P(X = 0)$ $\left[e^{-2} = 0 \cdot 1353 \right]$.

75. The mean life time of 100 fluorescent light bulbs produced by a company is computed to be 1570 with a standard deviation of 120 hours. If μ is the mean life time of all the bulbs produced by the company, test the hypothesis $\mu = 1600$ hours against the alternative hypothesis $\mu \neq 1600$ hours using 5% level of significance.

76. A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertisement campaign, the mean sales per week per shop was 140 dozens. After the campaign, a sample of 26 shops was taken and the mean sales was found to be 147 dozens with standard deviation 16. Can you consider the advertisement effective ?

77. Calculate three yearly moving average of the following data :

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
No. of Students	15	18	17	20	23	25	29	33	36	40

78. Out of 3000 unskilled workers of a factory, 2000 come from rural area and out of 1200 skilled workers 300 come from rural area. Determine the association between skill and residence.

79. Consider the following pay-off (profit) matrix. No probabilities are known for the occurrence of the nature states.

Action	States			
	S_1	S_2	S_3	S_4
A_1	5	10	18	25
A_2	8	7	8	23
A_3	21	18	12	21
A_4	30	22	19	15

Compare the solutions obtained by

i) Maximin   ii) Laplace.

PART – IV

Note : i) Answer any *four* questions.

ii) Each question carries *ten* marks.   4 × 10 = 40

80. In a bolt factory machines A_1, A_2, A_3 manufacture respectively 25%, 35% and 40% of the total output. Of these, 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine A_2 ?

81. The weekly remunerations paid to 100 lecturers coaching for professional extrance examinations are normally distributed with mean Rs. 700 and standard deviation Rs. 50. Estimate the number of lecturers whose remuneration will be (i) between Rs. 700 and Rs. 720, (ii) more than Rs. 750, (iii) less than Rs. 630.

82. In a certain city 125 men in a sample of 500 are found to be self employed. In another city the number of self-employeds is 375 in a random sample of 1000. Does this indicate that there is a greater population of self-employeds in the second city than in the first ?

83. An IQ test was administered to 5 persons before and after they were trained. The results are given below :

Candidates	I	II	III	IV	V
IQ before training	110	120	123	132	125
IQ after training	120	118	125	136	121

Test whether there is any change in IQ after the training programme (test at 1% level of significance).

84. Three varieties of coal were analysed by four chemists and the ash-content in the varieties was found to be as under :

Varieties	Chemists			
	1	2	3	4
A	8	5	5	7
B	7	6	4	4
C	3	6	5	4

Carry out the analysis of variance.

85. Fit a straight line trend by the method of least squares to the following data.

Find the trend values and the estimate production for the year 2002 :

Year :	1994	1995	1996	1997	1998	1999	2000
Production in Tonnes	80	90	92	83	94	99	92

86. A shop-keeper has the facility to store a large number of perishable items. He buys them at a rate of Rs. 3 per item and sells at the rate of Rs. 5 per item. If an item is not sold at the end of the day, then there is a loss of Rs. 3 per item. The daily demand has the following probability distribution :

Number of items demanded	3	4	5	6
Probability	0·2	0·3	0·3	0·2

How many items should he store so that his daily expected profit is maximum ?