MATHEMATICS PAPER - I

Time : 2 Hrs.) **Question Paper : October 2010** **(Max. Marks : 40**

Q. 1. (A) Attempt any TWO of the following : **(6)**

(i) State the truth value of the following propositions :

(a) The smallest prime number is 1. (b) The square of an odd integer is odd.

(c) A quadratic equation cannot have more than two roots.

(ii) By constructing a truth table, verify whether the following statement pattern is a tautology, a contradiction or a contingency : $(p \wedge (p \to \sim q)] \to q$

(iii) Construct the switching circuit for the following logical statement :

$(p \vee \sim q) \vee (q \wedge r)$

(B) Attempt any ONE of the following : **(2)**

(i) A diet is to contain at least 80 units of vitamin A and 100 units for minerals. Two foods F_1 and F_2 are avialabel. Food F_1 costs Rs. 4 per gram and Food F_2 costs Rs. 5 per gram. One gram of food F_1 contains minimum 3 units of vitamin A and 4 units of minerals. One gram of F_2 contains minimum 6 units of vitamin A and 3 units of minerals. Formulate this as L.P.P. to minimize the cost of diet.

(ii) Draw a graph of the following inequalities :

$2x + 2y \geq 12$, $5x + y \geq 10$, $x + 4y \geq 12$, $x \geq 0$, $y \geq 0$.

State only the vertices of the feasible region.

Q. 2. (A) Attempt any TWO of the following : **(3)**

(i) If $\bar{a}, \bar{b}, \bar{c}$ are the three non-zero non-coplanar vectors, then prove that any vector $\bar{r}$ in space can be uniquely expressed as a linear combination of the vectors $\bar{a}, \bar{b}, \bar{c}$ as $\bar{r} = x\bar{a} + y\bar{b} + z\bar{c}$, where x, y, z are all non-zero scalars.

(ii) By vector method prove that "medians of a triangle are concurrent."

(iii) Show that points A, B, C, D are coplanar, where $A \equiv (2, 3, 5), B \equiv (1, 1, 8), C \equiv (5, 4, 1), D \equiv (2, 2, 6)$.

(B) Attempt any ONE of the following : **(2)**

(i) If $\bar{a} = \bar{i} + 2\bar{j} + \bar{b} = 3\bar{i} + \bar{k}, \bar{c} = \bar{j} - \bar{k}$ *find* $\bar{a}(\bar{b} \times \bar{c})$

(ii) ABCDE is a pentagon, show that $\overline{AB} + \overline{AE} + \overline{BC} + \overline{DC} + \overline{ED} = 2\overline{AC}$

Q. 3. (A) (a) Attempt any ONE of the following : **(3)**

(i) Find the inverse of the following matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ by using Adjoint method.

(ii) Solve the following equations by reduction method.
$x + y + z = 9;\ 2x + 5y + 7z = 52;\ 2x + y - z = 01$

(b) Attempt any ONE of the following : **(3)**

(i) Show that every homogeneous equation of second degree in x and y represents a pair of straight lines passing through the origin.

(ii) Find the condition that the line $y = mx + c$ is tangent to the circle, $x^2 + y^2 = a^2$.

(B) Attempt any ONE of the following : **(2)**

(i) Find the combined equation of the pair of the lines through the origin such that one of them is parallel to $3x - y = 7$ and other is perpendicular to $2x + y = 8$.

(ii) Find the equation of the circle having (-1,2) and (3,-4) as the end point of diameter.

Q. 4. (A) (a) Attempt any ONE of the following : **(3)**

(i) If the angle between the lines $ax^2 + 2hxy + by^2 = 0$ is equal to the angle $100(h^2 - ab) = (a + b)^2$.

(ii) A circle cuts off an intercept of 6 units from the line $3x - 4y - 2 = 0$. If the centre of the centre of the circle is (-2,3), find the equation of the circle.

(b) Attempt any ONE of the following : **(3)**

(i) Two cards are drawn at random from a well shuffled pack of 52 cards. Find the probability that the cards drawn contain one heart card and the other spade card.

(ii) If A and B are any two events of a sample space S, then prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(B) Attempt any ONE of the following : **(2)**

(i) Find the equation of the normal to hyperbola $x^2 - 4y^2 = 36$ at a point (10,4).

(ii) Find the eccentricity and length of the latus rectum of the ellipse, $3x^2 + 4y^2 = 1$.

Q. 5. (A) (a) Attempt any ONE of the following : **(3)**

(i) Find equation of the ellipse in the standard form whose distance between foci is 6 and eccentricity is 3/5.

(ii) Show that two tangents drawn from the point (-6,9) to the parabola, $y^2 = 24x$ are at right angles.

(b) Attempt any ONE of the following : **(3)**

(i) Find the vector equation and Cartesian equation of the line passing through the two points (1,-2,1) and (0,-2,3).

(ii) Find the angle between the planes : $\bar{r}\left(2\bar{i} - \bar{j} + \bar{k}\right) = 6$ and $\bar{r}\left(\bar{i} + \bar{j} + 2\bar{k}\right) = 7$.

(B) Attempt any ONE of the following : **(2)**

(i) If e_1 and e_2 are the eccentricities of the hyperbolas, $(x^2/a^2) - (y^2/b^2) = 1$ and $(y^2/b^2) - (x^2/a^2) = 1$ respectively. Then prove that : $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

(ii) Find the Cartesian co-ordinates of the point on the parabola $y^2 = 8x$ whose parameter is 2.

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