Total No. of Questions : 12] [Total No. of Printed Pages : 7

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F. E. Examination - 2010

ENGINEERING MATHEMATICS - II

(2003 Course)

Time : 3 Hours] **[Max. Marks : 100**

Instructions :

(1) Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6 from section I and Q. No. 7 or 8, Q. No. 9 or 10, Q. No. 11 or 12 from section II.

*(2) Answers to the **two sections** should be written in **separate answer-books**.*

(3) Figures to the right indicate full marks.

(4) Neat diagrams must be drawn wherever necessary.

(5) Use of electronic pocket calculator is allowed.

(6) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Form the differential equation for which $y = ae^{3x} + be^{x}$ is the solution. **[05]**

(B) Solve **any three** : **[12]**

(1) $(x + 2y)(dx - dy) = dx + dy$

(2) $(e^{y} + 1) \cos x \, dx + e^{y} \sin x \, dy = 0$

(3) $(y^{3} - 2x^{2}y) \, dx + (2xy^{2} - x^{3}) \, dy = 0$

(4) $x \dfrac{dy}{dx} + y \log y = xye^{x}$

OR

Q.2) (A) Form a differential equation for which $y = A \cos x + B \sin x$ is the solution. **[05]**

(B) Solve **any three** : **[12]**

(1) $x^4 \frac{dy}{dx} + x^3y - \sec(xy) = 0$

(2) $(2xy + x^2) \frac{dy}{dx} = 3y^2 + 2xy$

(3) $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$

(4) $(2x \log x - xy)\, dy + 2y\, dx = 0$

Q.3) Solve **any three** :

(a) Find orthogonal trajectories of $r = a(1 - \cos\theta)$ **[05]**

(b) A moving body is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2 where x and v are displacement and velocity of the particle at that instant. Find the velocity of the particle in terms of x, if it starts from rest. **[06]**

(c) Uranium disintegrates at a rate proportional to the amount present at any instant. If m_1 and m_2 grams of uranium are present at time t_1 and t_2 respectively show that half life of uranium is

$$\frac{(t_1 - t_2) \log 2}{\log \frac{m_1}{m_2}}.$$ **[05]**

(d) Solve the equation $L\frac{di}{dt} + Ri = E_o \sin\omega t$ where L, R and E_o are constants and discuss the case when t increases indefinitely. **[06]**

OR

Q.4) Solve **any three** :

(a) The rate at which a body cools is proportional to the difference between the temperature of the body and that of surrounding air. If a body in air at 25°C will cool from 100°C to 75°C in one minute, find its temperature at the end of 3 minutes. **[05]**

(b) A particle falls in a vertical line under gravity and the force of air resistance to its motion is proportional to its velocity. Show that its velocity cannot exceed a particular limit. **[06]**

(c) When switch is closed in a circuit containing a battery E, a resistance R and an inductance L, the current i builds up at a rate given by $L\frac{di}{dt} + Ri = E$. Find i as a function of t.

How long will t be, before the current has reached one half its maximum value, if E = 6V, R = 100Ω and L = 0.1 henry ? **[05]**

(d) Under certain conditions, cane sugar is converted into dextrose at a rate, which is proportional to the amount unconverted at any time. If out of 75 gms of Sugar at t = 0, 8 gms are converted during the first 30 minutes, find the amount converted in $1\frac{1}{2}$ hours. **[06]**

Q.5) (A) Find the equation of the sphere which touches the sphere $4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$ at the point (1, 2, –2) and passes through (–1, 0, 0). **[06]**

(B) Find the equation of right circular cone which passes through the point (1, 1, 2) and has its axis as the line $6x = -3y = 4z$ and vertex as origin. **[05]**

(C) Find the equation of right circular cylinder whose axis is $x = 2y = -z$ and radius is 4. **[05]**

OR

Q.6) (A) Prove that the circles

$$x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0;\ 5y + 6z + 1 = 0$$

$$x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0;\ x + 2y - 7z = 0$$

lie on the same sphere and find its equation. **[06]**

(B) Find the equation of right circular cone which has its vertex at the point (0, 0, 12), whose intersection with the plane $z = 0$ is a circle of diameter 10. **[05]**

(C) Find the equation of right circular cylinder whose generator passes through (0, 0, 5) and axis passes through (1, 1, 3) and is perpendicular to z-axis. **[05]**

SECTION - II

Q.7) (A) Expand $f(x) = x - x^2$, $0 < x < 1$ in a half range (i) cosine series, (ii) sine series. Hence deduce from sine series that

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - - - - - - - - - - - \frac{\pi^2}{32}$$ **[08]**

(B) Show that $\int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}}\, dy = B(m, n)$ **[04]**

(C) If $I_n = \int_0^{\pi/4} \frac{\sin(2n-1)x}{\sin x} dx$, then prove that

$n(I_{n+1} - I_n) = \sin \frac{n\pi}{2}$ and hence find I_3. **[05]**

OR

Q.8) (A) Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y as given in the following table. Also obtain amplitude of the first harmonic. **[08]**

x	0	1	2	3	4	5
y	07	16	22	24	26	18

(B) Evaluate $\int_0^{\infty} \sqrt[4]{x} \cdot e^{-\sqrt{x}} dx$ **[04]**

(C) If $I_n = \int_0^{\pi/2} x \cos^n x \, dx$, obtain the relation between I_n and I_{n-2}. Hence find I_4. **[05]**

Q.9) (A) Prove that $\int_0^1 \frac{x^a - 1}{\log x} dx = \log(a+1),\ a > 0$. **[05]**

(B) Trace the following curves : **(Any Two)** **[08]**

(1) $x(x^2 + y^2) = a(x^2 - y^2),\ a > 0$

(2) $x = a(t + \sin t),\ y = a(1 + \cos t)$

(3) $r = 2a\cos\theta$

(C) Prove that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2} [\text{erf}(b) - \text{erf}(a)]$ **[04]**

OR

Q.10) (A) If $f(x) = \int_0^x (x-t)^2 \, G(t) \, dt$ then

prove that $\dfrac{d^3f}{dx^3} = 2 \, G(x)$. **[04]**

(B) Trace the following curves : **(Any Two)** **[08]**

(1) $a^2y^2 = x^2 (a^2 - x^2)$

(2) $r = a (1 + \cos\theta)$

(3) $r = a \sin 2\theta$

(C) Find the arc length of the cycloid $x = a (\theta + \sin\theta)$, $y = a (1 - \cos\theta)$ from cusp $\theta = -\pi$ to another cusp $\theta = \pi$. **[05]**

Q.11) (A) Evaluate $\iint_R \sqrt{xy (2-x-y)} \, dxdy$ where R is the area bounded by $x = 0, \ y = 0, \ x + y = 2$. **[05]**

(B) Evaluate $\int_0^\infty \int_0^\infty \int_0^\infty \dfrac{dxdydz}{(1+x^2+y^2+z^2)^2}$ **[05]**

(C) Find the centroid of the region in the first quadrant bounded by $\dfrac{x}{2} + \dfrac{y}{3} = 1$. **[06]**

OR

Q.12) (A) Find the total area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ **[05]**

(B) Find the volume common to cylinders
$x^2 + y^2 = a^2$, $x^2 + z^2 = a^2$ **[05]**

(C) Find the moment of inertia about the line $\theta = \frac{\pi}{2}$ of the area enclosed by $r = a\ (1 + \cos\theta)$. **[06]**