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UNIVERSITY OF HYDERABAD

Entrance Examination, 2005

Ph.D. (Statistics-OR)

Hall Ticket No.							

Time: 2 hours

Max. Marks: 75
Part-A: 25
Part-B: 50

Instructions

1. Calculators are not allowed.
2. Part-A carries 25 marks. Each right answer carries **1 mark** and each wrong answer carries - **1/4 mark**. So do not gamble. If you want to change any answer, cross out the old one and circle the new one. Over written answers will be ignored.
3. Part-B carries 50 marks. Instructions for answering Part-B are given at the beginning of Part-B.
4. Use the booklet provided for Part-B.
5. Do not detach any pages from the booklet.

Answer Part-A by **circling** the correct letter in the array below:

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e
21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e

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PART-A

Find the correct answer and mark it on the answer box on the previous page. A correct answer gets **1 mark** and a wrong answer gets $-\frac{1}{4}$ **mark.**

1) A,B,C are non-null mutually independent events. $P(AB|C)$ is equal to

(a) $\frac{P(A)\,P(B)}{P(C)}$

(b) $P(A|B)$

(c) $P(A)\,P(B)$

(d) $P(A|BC)$

(e) none of the above

2) A SRSWOR of size 2 is drawn from a population of size 10, whose values are 1,...,10. The probability that the population mean is estimated to be less than or equal to 4 is

(a) $12 / \binom{10}{2}$

(b) $3 / \binom{10}{2}$

(c) $9 / \binom{10}{2}$

(d) $6 / \binom{10}{2}$

(e) none of the above

3) $\Omega = [0, 2]$ and $P((a,b]) = \frac{b-a}{2}$ $0 \le a \le b \le 2$ define a sequence of functions $\{f_n\}$ as follows $f_n(w) = \begin{cases} n & w \in (0, \frac{1}{n}] \\ 0 & 0.w \end{cases}$

$\{f_n\}$

(a) does not converge to 0 in probability

(b) converges to 0 in probability

(c) converges to 0 in mean

(d) converges to 0 in mean square

(e) none of the above

4) $\{A_n\}$ is a sequence of subsets of $(-1, 1)$, $A_n = \left[-\frac{1}{2^{n+1}}, \frac{1}{2^n}\right]$, $\limsup_{n \to \infty} A_n$ is equal to

(a) $(-\frac{1}{2}, \frac{1}{2}]$

(b) ϕ

(c) $\{1\}$

(d) $\{0\}$

(e) none of the above

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5) $X \sim P(\lambda)$ an unbiased estimator for $e^{2\lambda}$ based on one observation X is

(a) 3^X

(b) 2^X

(c) 2^{2X}

(d) does not exist

(e) none of the above

6) For the random variable X with probability distribution:

$P\left[X = j\right] = \frac{4}{j(j+1)(j+2)}$, j = 1,2,...

(a) EX^r does not exist for any r = 1,2,...

(b) V(X) exists

(c) $EX^{2r+1} = 0$ for all r = 0,1,2,...

(d) EX^r exists for all r = 1,2,...

(e) none of the above

7) $X_j \sim U(0, j) = j = 1,2,\dots,n$ and are independent $-\log \frac{\prod_{i=1}^{n} X_i}{n!}$ has pdf

(a) e^{-x} for $x \geq 0$

(b) $\frac{x^{n-1}e^{-x}}{\Gamma n}$ for $x \geq 0$

(c) xe^{-x} for $x \geq 0$

(d) $\frac{x^n e^{-x}}{\Gamma n}$ for $x \geq 0$

(e) none of the above

8) Which of the following could be the characteristic function of the random variable X whose pdf is $\frac{1}{2}e^{-|x|}$ $x \in \mathbb{R}$?

(a) 1 / (1-it)

(b) 2 / (2-it)

(c) $1 / (1-it)^2$

(d) $4 / (2-it)^2$

(e) none of the above

9) $X_n \sim G(n, 2)$, then $\lim_{n \to \infty} P(X_n \geq n/2)$

(a) is equal to 0

(b) is equal to 1

(c) is equal to 1/2

(d) does not exist

(e) none of the above is correct

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10) The quality characteristic is monitored by a control chart designed so that the probability that a certain out of control condition will be detected in the first sample following the shift is $1-\beta$. The expected number of samples analysed before the shift is detected is

(a) $1/\beta$

(b) $1 / (1-\beta)$

(c) $1 / \beta^2$

(d) $1 / (1-\beta^2)$

(e) none of the above

11) $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 & 1/2 \\ 1/2 & 2 \end{pmatrix}\right)$ $Y_1 = X_1+X_2$ and $Y_2 = aX_1+bX_2$ are independent if and only if

(a) $a+b = 0$

(b) $2a+3b = 0$

(c) $2a+4b = 0$

(d) $3a+5b = 0$

(e) none of the above

12) Based on the random sample $X_1,\ldots,X_n$ from $N(\mu, \sigma^2)$ an unbiased estimator for μ^2

(a) is $\frac{1}{n} \sum_{i=1}^{n} X_i^2$

(b) is $\bar{X}^2$

(c) does not exist

(d) is $-\frac{1}{n(n-1)} \left(\sum_{i=1}^{n} X_i^2 - n^2\bar{X}^2\right)$

(e) is none of the above

13) $X_1,X_2,\ldots,X_n$ is a random sample from the $N(\sqrt{\theta}, \theta)$ population. Then

(a) $\bar{X}$ is a sufficient statistic for θ

(b) $(\bar{X}, \sum_{i=1}^{n} (X_i-\bar{X})^2)$ is sufficient but not complete

(c) $(\bar{X}, \sum_{i=}^{n} (X_i-\bar{X})^2)$ is not sufficient for θ

(d) $\sum_{i=1}^{n} (X_i-\bar{X})^2$ is sufficient for θ

(e) none of the above

14) The p-value for testing a certain null hypothesis (H_0) is 0.025.

(a) H_0 will be rejected at any level of significant (l.s)

(b) H_0 will be accepted at 0.1 l.s.

(c) H_0 will be rejected at .05 l.s. but will be accepted at 0.01 l.s.

(d) H_0 will never be rejected

(e) none of the above is correct

15) $X_1, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$ which of the following is the pivotal quantity in the $(1-\alpha)100\%$ confidence interval of μ?

(a) $Q_1 = \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

(b) $Q_2 = \dfrac{\bar{X}-\mu}{S/\sqrt{n}}$

(c) $Q_3 = \bar{X} - \mu$ where $S^2 = \dfrac{1}{n-1} \sum_{i=1}^{n} (X_i-\bar{X})^2$

(d) $Q_4 = \dfrac{\bar{X}-\mu}{S/\sqrt{n-1}}$

(e) none of the above

16) $\alpha_1, \alpha_2, \alpha_3$ are the unknown weights of 3 objects, the following measurements were made:

$$\alpha_1+\alpha_2+\alpha_3 = 15; \quad \alpha_1+\alpha_2 = \alpha_3+3; \quad \alpha_1+\alpha_3 = 6+\alpha_2$$

assume that these observations were recorded with uncorrelated, homoscedstic errors:

(a) $\alpha_1, \alpha_2, \alpha_3$ are all estimable

(b) α_1 is estimaable but α_2 and α_3 are not estimable

(c) α_2 is estimable but α_1 and α_3 are not estimable

(d) none of $\alpha_1, \alpha_2, \alpha_3$ is estimable

(e) none of the above is correct

17) Consider the simple linear regression Y = a+bX+c let $\hat{Y}$ be the prediction of Y given X. The correlation coefficient between Y and $\hat{Y}$ is

(a) always 0

(b) always negative

(c) always positive

(d) always 1

(e) none of the above

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18) In the 2^4 factorial design, which of the factors or interactions are confounded with the following blocks.

Block-1	0000	1001	1010	1100	0011	0101	0110	1111
Block-2	1000	0001	0010	0100	1011	1101	1110	0111

(a) AB

(b) AC

(c) ABC

(d) ABCD

(e) none of the above

19) A population with two strata has sizes N and 2N and their variances are $2S^2$ and S^2 respectively. If the costs of drawing units is the same from each stratum, the optimal allocation (n_1, n_2) for estimating population mean for a tototal sample size n is

(a) $n_1 = n_2$

(b) $n_1 = 2n_2$

(c) $n_1 = n_2/2$

(d) $n_1 = \sqrt{2}\, n_2$

(e) none of the above

20) The reliability function R(x) of a component is $e^{-\lambda x^{\beta}}$ ($\lambda>0$, $\beta>0$ $x\geq0$) has increasing failure rate (IFR) if

(a) $\lambda > 1$

(b) $\beta > 1$

(c) $\lambda > 1$

(d) $\beta = 1$

(e) $\beta < 1$

21) $A = (0,2] \cup [3,6)$; $B = [1,4]$ then $A \Delta B$ is

(a) an open interval

(b) a closed set

(c) an open set

(d) a connected set

(e) none of the above

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22) Consider the following system of equations:

$$x_1 + x_2 - x_3 = 9$$
$$2x_1 + 3x_3 = \alpha$$
$$3x_1 + x_2 + 2x_3 = 18$$

What should α be so that this system is consistent?

(a) 9

(b) 10

(c) 3

(d) 2

(e) none of the above

23) A is a nxn skew symmetric real matrix that is $a_{ji} = -a_{ij}$ $\quad i = 1,\dots,n$; $j = 1,\dots,n$ then,

(a) all the characteristic roots of A are positive

(b) all the characteristic roots of A are reals

(c) all the non-zero characteristic roots of A are complex

(d) some characteristic roots are positve and the rest are complex

(e) none of the above

24) $\lim_{n \to \infty} \frac{1}{n^{2k+1}} \sum_{j=1}^{n} j^{2k}$ is

(a) 1 / 2(k+1)

(b) 1 / 2k+1

(c) 1 / k+1

(d) 0

(e) none of the above

25) The function F is defined as follows:

$$F(x) = \begin{cases} 0 & x < 0 \\ \sum_{r=0}^{[x]} \frac{e^{-r}}{r!} & x \geq 0 \end{cases}$$

where [x] = largest integer less than or equal to x

F is

(a) continuous everywhere

(b) continuous everywhere except at x = 0

(c) not continuous anywhere

(d) not continuous at x = 0,1,2,...

(e) none of the above

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PART-B

There are 15 questions in this part. Each question carries 10 marks. Answer as much as you can, the maximum you can score is 50marks. The answers should be written in the separte answer booklet supplied to you.

1) The distribution function of a random variable X is as follows:

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x/4 & 0 \le x < 1 \\ 1/3 & 1 \le x < 2 \\ 1/2 & 2 \le x < 3 \\ 2/3 & 3 \le x < 4 \\ \frac{x-2}{3} & 4 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

(a) Trace the graph of F_X.

(b) Identify (if any) the points of discontinuity of F_X.

(c) Evaluate (i) $\Pr\{1.5 \le X \le 3.5\}$ (ii) the median of X.

(d) Evaluate E(X).

2) (a) Let $\{X_n\}$ be a sequence of independent exponential random variables with mean $\lambda = 1$. Prove that

$$\left(\prod_{k=1}^{n} X_k\right)^{1/n} \xrightarrow{P} e^{-C} \text{ as } n \longrightarrow \infty \text{ where } C = -\int_0^\infty \log x \, e^{-x} dx.$$

(b) $X_i \sim P(1)$ $i = 1,2,\ldots$ and are independent let $T_n = X_1+2X_2+3X_3+\ldots nX_n$ determine the limiting distribution of $\frac{T_n - ET_n}{\sqrt{VT_n}}$ (with proofs and stating results used).

3) (a) $X_1,\ldots,X_n$ are iid random variables for $m \le n$, evaluate

$$E\left(\frac{\sum_{i=1}^{m} X_i}{\sum_{i=1}^{n} X_i}\right).$$

(b) $X_1,\ldots,X_n$ are iid continuous random variables. We say that a record value occurs at j, $j \le n$ if $X_j \ge X_i$ for $i = 1,\ldots,j-1$. Let R_n denote the number of record values upto n, evaluate $E(R_n)$.

4) $X_1, X_2, \ldots, X_n$ is a random sample from $U(-\theta, \theta)$ $(\theta>0)$

(a) Determine the MLE of θ.

(b) Is the MLE of θ a single sufficient statistic?

(c) Is it a complete sufficient statistic?

Answer with proofs wherever necessary.

5) $X_1, \ldots, X_n$ is a random sample from the $\exp(\lambda)$ population.

(a) Obtain the MVUE for λ.

(b) Obtain the conditional distribution of X_1 given X_1+X_2.

6) (a) Obtain the most powerful level α test

$$\text{for } H_0:\ f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \qquad -\infty<x<\infty$$

$$H_1:\ f(x) = \frac{e^{-|x|}}{2} \qquad -\infty<x<\infty$$

based on one observation.

(b) Write down the power function for the above test.

7) $$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N\left(\begin{pmatrix} \mu \\ \mu \\ \mu \end{pmatrix}, \sigma^2\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}\right)$$

Evaluate (i) $E(X_1 | X_1+X_2)$

(ii) Obtain the BLUE of μ.

8) In order to compare the effects of 4 treatments A,B,C and D, a block design of three blocks, each of which consists of 2 plots was used. Treatments A and D were given to block-1, B and C to block-2 and A and C to block-3.

(a) Specify with justification whether the design is (i) balanced (ii) connected.

(b) with justification state how many orthogonal treatments contrasts are estimable.

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9) A population of N units consists of values $Y_1, \ldots, Y_N$ let T_1 be the estimator of the population total based on SRSWOR of size n. Alternatively, it is decided that the first population unit must be in the sample and the rest of the (n-1) units are to be selected by SRSWOR from the remaining (N-1) units. Let $T_2 = Y_1 + (N-1)\bar{y}_{n-1}$ be the new estimator of the population total ($\bar{y}_{n-1}$ is the mean of the (n-1) sampled units).

(a) Verify whether T_2 is an unbiased estimator for population total.

(b) Determine the variance of T_2.

(c) Express the variance of T_2 in terms of that of T_1.

10) Consider a population of 100 units taking values $Y_1, \ldots, Y_{100}$. To estimate the population mean a SRSWOR of 10 units is drawn and then the mean, of 2 units drawn by SRSWOR from these 10 units is used to estimate the population mean. Evaluate the expected value and variance of this estimator.

11) The Markov chain $\{X_i,\ i \geq 1\}$ with state space $\mathscr{S} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ has the transition probabilities

$$P_{m,n} = P_r\{X_{i+1} = n \mid X_i = m\} = \frac{1}{2} \quad \text{if } n = m+1 \text{ or } m-1$$
$$= 0 \quad \text{O.w.}$$

(a) (i) Is this Markov chain irreducible?

(ii) Determine its recurrent states (if any).

(b) Does it have a (i) stationary distribution

(ii) limiting distribution?

If yes determine them (whichever exists). Otherwise explain why?

12) $X_1, \ldots, X_n$ is a random sample from the exp(1) population. Let $X_{(1)}, \ldots, X_{(n)}$ be the ordered sample, show that

$X_{(1)} \sim \exp(n)$

$X_{(i)} - X_{(i-1)} \sim \exp(n-i+1),\ i = 2, 3, \ldots, n$

Thus determine the distribution of $X_{(2)}$ and evaluate $E(X_{(r)})$.

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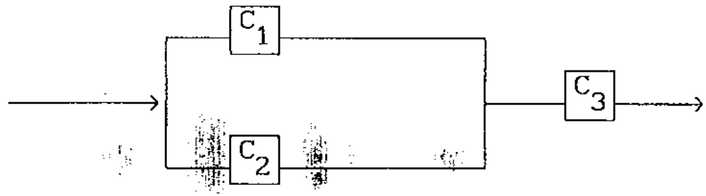
13) Three independently-functioning components are connected into a single system as in the figure.



Suppose that the reliability of each of the components for an operational period of x hours given by $R(x) = e^{-0.03x}$. If T is the time to failure of the system

(a) Determine the pdf of T

(b) Obtain the reliability function of the system?

(c) How does it compare with $e^{-0.03x}$.

14) Write the dual for the following Linear Programming Problem and solve it grahically. Write the simplex table for the optimal solution obtained by graphical method and hence determine the optimal solution of the primal.

maximize $W = 3x + 7y + 2z$ subject to

$$-2x + 2y + z \leq 10$$
$$3x + y - z \leq 20$$
$$x, y \text{ and } z \geq 0$$

15) In a department courses C1, C2, C3, C4, C5 have to be offered for a given semester for a given batch. In the department there are four faculty members F1, F2, F3 and F4. The evaluation of each faculty for each of the 5 courses on a scale of 0-100 are given below:

	C1	C2	C3	C4	C5
F1	100	90	75	30	60
F2	100	80	80	10	20
F3	90	70	80	60	30
F4	80	60	50	30	10

(a) Suggest an optimal allocation so that teaching will be effective in that semester.

(b) What is the best allocation if F4 is not willing to teach any course other than C1? What is the percentage of loss?

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